1. (10%) Let $f(x) = \int_{\sqrt{x}}^{\tan^{-1} x} \dfrac{1}{t^6 + 1} dt$. Find $f'(x)$.

2. (20%) Let $f(x) = e^{-\frac{x^2}{2}}$

   (a) (12%) Find the taylor expansion of the function $f(x)$ at $x = 0$. (In the expansion, you need to write down the general term.)

   (b) (8%) Use the first three nonzero terms in (a) to estimate $\int_{-2}^{2} f(x)dx$, ignoring the error term.

3. (20%)

   (a) (8%) Find $a, b, c \in \mathbb{R}$ such that $\dfrac{u - 3}{(u - 1)(u^2 + 1)} = \dfrac{a}{u - 1} + \dfrac{bu + c}{u^2 + 1}$ for all u.

   (b) (12%) Find $\displaystyle\int_{\ln 2}^{\ln 3} \dfrac{e^{2x} - 3e^x}{(e^x - 1)(e^{2x} + 1)} dx$

4. (10%) Find $\displaystyle\int_{0}^{\frac{\sqrt{2}}{2}} \dfrac{x^2}{(1 - x^2)^{\frac{3}{2}}} dx$.

5. (10%) Find $\displaystyle\int_{1}^{e} (\ln x)^2 dx$.

6. (10%) Compute $\displaystyle\lim_{x \to 0^+} (1 + \sin 4x)^{\cot x}$.

7. (10%) Given the curve $\Gamma : y = \dfrac{1}{2}(e^x + e^{-x})$, find the area of the bounded region enclosed by Γ, $y = 0$, $x = 0$, and $x = 1$.

8. (10%) The profile of a football resembles the ellipse $\dfrac{x^2}{196} + \dfrac{y^2}{81} = 1$. Find the football's volume.