

1. (15%) Find the local extreme values and saddle point(s)(if any) of the function  $x^2 e^{x \cos y}$ , where  $x \in \mathbb{R}$ ,  $-\frac{\pi}{2} < y < 2\pi$ .
2. (10%) Find the tangent plane to  $x^4 + y^4 + z^4 = 9xyz$  at  $(1, 1, 2)$ .
3. (15%) Find the absolute minimum value of the function  $\sqrt{x^2 + 4y^2}$  on the curve  $xy = 2$  by using Lagrange multiplier method.
4. (10%) Calculate  $I = \int_{-1}^1 \int_{-1}^1 (e^{x^2} \sin y + x^2 y^4) dx dy$ .
5. (15%) Evaluate  $I = \iint_T (x + y)^{10} dx dy$  where  $T$  is the triangle with vertices  $(0, 0), (1, 1), (2, 0)$
6. (10%) Compute the integrals  $I = \int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2 y^2 dx dy$ .
7. (10%) Calculate  $I = \int_0^1 \int_{\sqrt{y}}^1 e^{x^3} dx dy$ .
8. (15%) Let  $f(x, y) = \ln \sqrt{x^2 + y^2}$ .
  - (a) Find the directional derivative of  $f$  at the point  $(3, 4)$  in the direction to point  $(2, 6)$ .
  - (b) At what direction is the function  $f$  changing fastest at the point  $(3, 4)$ ? What is the rate of change along this direction?