1. (13%) Find the derivative of the functions.

   (a) (5%) $\ln\left(\sqrt{1-x^2}-x\right)$.

   (b) (8%) $(\tan x)^{\sin x}$.

2. (10%) Evaluate

   (a) (5%) $\lim_{x\to 0} x\left(\cos x+\cos\frac{1}{x}\right)$.

   (b) (5%) $\lim_{x\to 1}\frac{\frac{x}{\sqrt{x^2+1}}-\frac{\sqrt{2}}{2}}{x-1}$.

3. (8%) Let $f(x)=\frac{(x+1)(x^3+x^2+1)(x^2-x+1)}{e^{\frac{x+1}{x+2}}\sqrt{x^2+1}}$. Find $f'(-1)$.

4. (12%) Let $\frac{\sqrt{3}}{2}+xy=\sin y$.

   (a) (6%) Find the equation of the tangent line at $(0,\frac{\pi}{3})$.

   (b) (6%) Find $\frac{d^2y}{dx^2}$ and evaluate at $(0,\frac{\pi}{3})$.

5. (12%) Let $f(x)=x^3+x+\cos x$, $x\in\mathbb{R}$.

   (a) (6%) Show that $f(x)$ is a one-to-one function.

   (b) (6%) Let $g(x)$ be the inverse function of $f(x)$. Find $g'(1)$.

6. (10%) Show that $y=1-x$ and $y=\cos x$ intersect at only one point.

7. (25%) Let $y=f(x)=\frac{x(x-1)+2}{x+1}$. Find the following

   (a) the intervals on which $y=f(x)$ increases ______________________________

   the intervals on which $y=f(x)$ decreases ______________________________ (6%)

   (b) the intervals on which $y=f(x)$ is concave up ______________________________

   the intervals on which $y=f(x)$ is concave down ______________________________ (6%)

   (c) the local maximum(if exists) of $y=f(x)$: ______________________________ (coordinates)

   the local minimum(if exists) of $y=f(x)$: ______________________________ (coordinates) (6%)

   (d) all asymptotes of $y=f(x)$ ______________________________ (4%)

   (e) Sketch the graph of $y=f(x)$. (3%)

8. (10%) A boat leaves a dock at 5 : 00 PM and travels due north at a speed of 20 km/h. Another boat has been heading due west at 15 km/h and reaches the same dock at 6 : 00 PM. At what time were the two boats closest together?