

1. (15%) On the curve $y^3 + xy^2 + x^2y - 2x^3 = 1$, use implicit differentiation to find (a) $\frac{dy}{dx}$ and (b) the values of $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ at the point $x = 1, y = 1$.
2. (12%) Find the point on $y^2 = 2x$ that is closest to the point $(1, 4)$.
3. (21%) Let $y = f(x) = \frac{x^3}{1+x^2}$. Answer the following questions. Fill each blank and give your reasons (and computations). Put **None** in the blank if the item asked does **not** exist.

(a) The function is increasing on the interval(s) _____ (5%).

The local maximal point(s) $(x, y) =$ _____. The local minimal point(s) $(x, y) =$ _____. (2% total) **Reason:**

(b) The function is concave upward on the interval(s) _____ and concave downward

on the interval(s) _____. (5% total)

The inflection point(s) $(x, y) =$ _____ (2%). **Reason:** (c) The asymptote lines of the function are _____ (3%). **Reason:**

(d) Sketch the graph of the function. Indicate, if any, where it is increasing/decreasing, where it concave upward/downward, all relative maxima/minima, inflection points and asymptotic line(s) (if any). (4%)

4. (10%) (a) Find the linear approximation of $(1+x)^{1/3}$ at $x = 0$.

(b) Use (a) to estimate $\sqrt[3]{8.03}$.

5. (12%) Let $f(x) = 3x - \sin x$.

(a) Prove that $f(x)$ is an increasing function.

(b) Let $g(x)$ be the inverse function of $f(x)$. Find $g'(0)$.

6. (20%) Evaluate y' .

(a) $y = 3^{\cos x}$

(b) $y = \tan^{-1}(1+x^2)$

(c) $y = \ln(x + \sqrt{1+x^2})$

(d) $y = \frac{\cos x}{1+\sin x}$

7. (10%) Evaluate the following limits.

(a) $\lim_{x \rightarrow 1^+} \frac{\ln x}{x-1}$

(b) $\lim_{x \rightarrow 2} \frac{\sqrt{x^3+x^2-8}-2}{x-2}$