1. (15%) (a) (12%) Evaluate $\int \frac{dx}{x-1}$, $\int \frac{dx}{(x-1)^2}$, $\int \frac{xdx}{x^2+1}$ and $\int \frac{dx}{x^2+1}$. (3 points each)

   (b) (3%) Find real numbers A, B, C and D such that

$$\frac{4x-2x^2}{(x-1)^2(x^2+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{Cx+D}{x^2+1}.$$

   Then evaluate $\int \frac{4x-2x^2}{(x-1)^2(x^2+1)}dx$.

2. (10%) Derive the Taylor expansion of $f(x) = \cos^{-1} x$ at $x = 0$. Write down the explicit form of the general term. (You can use $\mathbf{C}_n^m$ in the answer)

3. (10%) Evaluate $\int x^n \ln x dx$, $n \geq 0$.

4. (8%) Let R be the region enclosed by $y = \frac{\sin x}{x}$, x-axis, $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$. Find the volume of the solid obtained by rotating R with respect to y-axis.

5. (10%) Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the x-axis.

6. (10%) Find the length of the curve $y = \frac{x^5}{6} + \frac{1}{10x^3}$, $1 \leq x \leq 2$.

7. (9%) Find the area of the bounded region enclosed by $x^2 = 2y$ and $y - x = 4$.

8. (8%) Evaluate the limit: $\lim_{x\to 0}(\cos x)^{\frac{1}{x^2}}$.

9. (12%) (a) Write down the Taylor expansion of $\ln(1-x)$ and $\tan^{-1} x$ at $x = 0$ with the explicit form of the general term. You don't have to derive them. (4 points each)

   (b) Evaluate the limit: $\lim_{x\to 0} \frac{\left(\ln(1-x)\right)\cdot\left(\tan^{-1} x\right) + x^2 + \frac{x^3}{2}}{x^5}$ (Hint. Use (a))

10. (8%) Evaluate $\int \sqrt{1-x^2}dx$.