1. (12%) Find an equation of the tangent plane to the surface $y - x = 4\arctan(xz)$ at the point $(1, 1, 0)$.

2. (12%) The temperature at a point (x, y) is given by $T(x, y) = 100e^{-x^2-3y^2}$, where T is measured in °C and x, y in meters.
   (a) In which direction does the temperature increase fastest at $P(1, -1)$?
   (b) What is this fastest increasing rate?
   (c) Evaluate the directional derivative $\frac{\partial T}{\partial \vec{u}}(1, -1)\Big|_{\vec{u}=(\frac{3}{5}, \frac{4}{5})}$

3. (12%) Evaluate $\int_0^1 \left(\int_{x^{\frac{1}{6}}}^1 \frac{1}{1+y^7} dy \right) dx$.

4. (12%) Find $I = \int\int_\Omega (x - 2y)^{3/2}(3x + y)^{1/2} dA$, where Ω is the region enclosed by $2x + 3y = 0$, $3x + y = 0$ and $x - 2y = 1$.

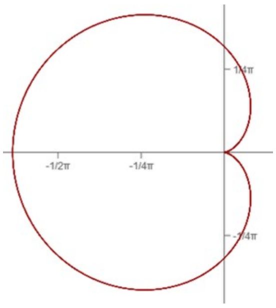
5. (15%) Find the critical points of $z = ye^{-\frac{1}{2}(x^2+y^2)}$, which give rise to local maxima? Local minima? Saddle points?

6. (10%) Let $f(x, y) = x^3y^5 - x^2y - y^3$ and $x = x(u, v)$, $y = y(u, v)$.
   Suppose $x(1, 3) = 2$, $y(1, 3) = 1$ and

$$\frac{\partial x}{\partial u}(1,3) = \frac{1}{5}, \qquad \frac{\partial x}{\partial v}(1,3) = \frac{1}{2}$$
$$\frac{\partial y}{\partial u}(1,3) = \frac{1}{11}, \qquad \frac{\partial y}{\partial v}(1,3) = \frac{1}{4}$$

   Find the value of $\frac{\partial f}{\partial u}$ and $\frac{\partial f}{\partial v}$ at $(u, v) = (1, 3)$.

7. (12%) Find $\iint_\Omega \sqrt{x^2 + y^2} dA$, where Ω is the region inside the cardioid $r = 1 - \cos\theta$.



8. (15%) Find the extremal values of $f(x, y) = x^2 + xy + y^2$ subject to the constraint $x^2 + 2xy + 2y^2 = 1$.