1. (25%) Let $y = \dfrac{x^2}{2} + \dfrac{1}{x}$. Answer the following questions. Fill each blank and give your reasons (and computations). Put **None** in the blank if the item asked does **not** exist.

   (a) The function is increasing on the interval(s)_____ and decreasing on the interval(s)_____(6% total). The local maximal point(s) $(x, y) =$ _____(2%), The local minimal point(s) $(x, y) =$_____(2%). **Reason:**

   (b) The function is concave upward on the interval(s)_____ and concave downward on the interval(s)_____(6% total).

   The inflection point(s) $(x, y) =$ _____(2%). **Reason:** (c) The asymptote lines of the function are _____(3%). **Reason:**

   (d) Sketch the graph of the function. Indicate, if any, where it is increasing/decreasing, where it concaves upward/downward, all relative maxima/minima, inflection points and asymptotic line(s) (if any).(4%)

2. (15%) Suppose $y = f(x)$ satisfy $\tan^{-1}\dfrac{y}{x} + \ln\dfrac{x^2+y^2}{2} = \dfrac{\pi}{4}$. Find y' and y'' in terms of x and y. Find their explicit values for $(x, y) = (1, 1)$.

3. (12%) Find the equation of the line tangent to $y = \dfrac{\cos x}{1+\sin x}$ at $x = \dfrac{\pi}{6}$.

4. (12%) Find $\lim\limits_{x\to 0}\dfrac{\cos x - 1}{x \sin x}$.

5. (12%) Find the linear approximation of $(128)^{\frac{1}{3}}$ (by a differential).

6. (12%) Let $g(x)$ be the inverse function of $f(x) = x^5 + 2x^3 + x - 2$. Find $g'(f(1))$.

7. (12%) Prove that $y = x$ and $y = \tan^{-1} x$ intersect at only one point.