1. (15%) Set $I_n = \int (\ln x)^n dx$, $n \geq 1$.

   (a) (6%) Find $I_1$.

   (b) (6%) Express $I_{n+1}$ in terms of $I_n$.

   (c) (3%) Use (b) to find $I_4$.

2. (12%) (a) (6%) Find $\displaystyle\int \frac{x+1}{x^2+x+1} dx$.

   (b) (6%) Find $\displaystyle\int \frac{dx}{e^x(e^{2x}-1)}$.

3. (12%) (a) (6%) Evaluate $\displaystyle\int_0^{\frac{1}{2}} x^2\sqrt{1-x^2}dx$.

   (b) (6%) Find $\displaystyle\frac{d}{dx}\int_x^{x^2} \frac{dt}{1+t^5}$.

4. (12%) (a) (6%) Evaluate $\int \sec^3\theta d\theta$. You can use the formula for $\int \sec\theta d\theta$ if you know it.

   (b) (6%) Find the arc length of the curve $y = \dfrac{x^2}{2} + 1$ from $x = 0$ to $x = 2$.

5. (10%) Find the volume of the solid obtained by rotating about the $y$-axis the region bounded by $x = 0$, $x = 1$, $y = 0$, and $y = \sqrt{x^2+1}$.

6. (15%)

   (a) (8%) Write down the fourth degree Taylor polynomial of $f(x) = \sin x$ at $x = 0$. Also, write down its remainder provided by Taylor's Theorem.

   (b) (7%) Find the numerical value of $\sin 20°$ accurate to within $10^{-4}$. Your answer can be expressed in terms of $\pi$. Don't have to bother replacing $\pi$ by $3.14\cdots$. But the remainder must be estimated in details to prove the asserted accuracy of your numerical value.

7. (12%) (a) (6%) Find $\displaystyle\lim_{x\to\infty} x^{\frac{1}{x}}$.

   (b) (6%) Find $\displaystyle\lim_{x\to 0}\frac{\ln(1+x^2)}{1-\cos x}$.

8. (12%) A rod is being carried horizontally down a hallway of 1m wide with a right-angled turn. What is the maximal length of the rod that can be carried around the corner?