1. (10%) Find the particular solution of $(t^2+1)y' = \frac{1}{y}$ determined by the initial condition $y(0) = 2$.

2. (10%) Find the particular solution of $y' - \frac{2}{t}y = t$ determined by the initial condition $y(1) = -1$.

3. (10%) Let V_2 be the random variable that means the second "+" event happening at the k-th Bernoulli trial. Compute the probability $P(V_2 = 10)$.
(Let p be the probability of getting "+" in one trial and $q = 1 - p$.)

4. (10%) Evaluate $\int_{-\infty}^{\infty} xe^{-(x-1)^2}dx$. $\left(\text{Assume that } \int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}\right)$

5. (15%) Let $f_X(t) = \begin{cases} \lambda e^{-\lambda t} & \text{if } t \geq 0 \\ 0 & \text{if } t < 0. \end{cases}$
Evaluate $E(X)$ and $Var(X)$.

6. The age of a randomly selected, alcohol-impaired driver in a fatal car crash is a random variable with probability density function given by

$$f(x) = \frac{c}{x^2}, \qquad 15 \leq x \leq 35.$$

   (a) (8%) Find c.
   (b) (7%) Calculate the probability of the age from 21 to 25 in such a fatal car crash.

7. The average number of accidents per year at a cross-roads is 36. (Note: a year = 360 days) Assume that the occurrences of accidents comply with Poisson distribution.
   (a) (8%) Calculate the probability that there is only one accident within 15 days.
   (b) (7%) Calculate the probability that there are two or more accidents within 15 days.

8. (15%) Let X, Y are random variables taking values 1 or 2. Assume

$$P(X=1,Y=1) = \frac{1}{14}, P(X=1,Y=2) = \frac{5}{14}, P(X=2,Y=1) = \frac{6}{14}, P(X=2,Y=2) = \frac{2}{14},$$

evaluate $P(X = 1)$, $P(X = 2)$ and $E(X)$ ∘ (each 5%)