1. (10%) Find $\displaystyle\lim_{x\to 2} \frac{\sqrt[3]{x^3 + x - 8} - \sqrt[3]{x}}{x - 2}$.

2. (10%) Use Mean Value Theorem to show $|\tan x - \tan y| \leq 2|x - y|$ for all $x, y \in [0, \pi/4]$.

3. (10%) Given the equation $xy^2 + x^2y - 2 = 0$. Find the equation of the tangent line at $(x, y) = (1, 1)$.

4. Let $f(x) = x^7 - 7x^6 + 5x^4$

   (a) (10%) Find the linear approximation of $f(x)$ at $x = 1$.

   (b) (5%) Estimate $f(0.92)$ using the linear approximation.

5. (15%) (5 points for each question) Differentiate the following function.

$$(1)\ \sin(3^x) \quad (2)\ e^{\tan^{-1} x} \quad (3)\ \frac{\ln x}{x}$$

6. (15%) Design a cylindrical cup with no cover (ignore the thicknesses of the cup). Assume that the volume of the cup is fixed to be 1000 cubic centimeter. Find the radius of the bottom of the cup such that the surface area for the cup is minimal(12%). The Extrema Test must be used for your answer(3%).

7. (25%) Let $y = f(x) = \dfrac{x(x - 2) + 2}{x - 1}$. Find the following if exists or applicable, and remark so if non-existent or inapplicable.

   (a) the intervals on which f increases _____

       the intervals on which f decrease _____

   (b) the local maximum of f _____(coordinates)

       the local minimum of f _____(coordinates)

   (c) the intervals on which f is concave up _____

       the intervals on which f is concave down _____

   (d) the points of inflection _____(coordinates)

   (e) all asymptotes _____

   (f) Sketch the graph of f.