Section 3.10 Linear Approximations and Differentials

34. Use a linear approximation (or differentials) to estimate the given number.

$$\sqrt{100.5}$$

Solution:

$y = f(x) = \sqrt{x} \quad \Rightarrow \quad dy = \frac{1}{2}x^{-1/2}\,dx$. When $x = 100$ and $dx = 0.5$, $dy = \frac{1}{2}(100)^{-1/2}\left(\frac{1}{2}\right) = \frac{1}{40}$, so

$\sqrt{100.5} = f(100.5) \approx f(100) + dy = 10 + \frac{1}{40} = 10.025$.

36. Use a linear approximation (or differentials) to estimate the given number.

$$\cos 29^\circ$$

Solution:

$y = f(x) = \cos x \quad \Rightarrow \quad dy = -\sin x\,dx$. When $x = 30^\circ$ $[\pi/6]$ and $dx = -1^\circ$ $[-\pi/180]$,

$dy = \left(-\sin\frac{\pi}{6}\right)\left(-\frac{\pi}{180}\right) = -\frac{1}{2}\left(-\frac{\pi}{180}\right) = \frac{\pi}{360}$, so $\cos 29^\circ = f(29^\circ) \approx f(30^\circ) + dy = \frac{1}{2}\sqrt{3} + \frac{\pi}{360} \approx 0.875$.

48. When blood flows along a blood vessel, the flux F (the volume of blood per unit time that flows past a given point) is proportional to the fourth power of the radius R of the blood vessel:

$$F = kR^4$$

(This is known as Poiseuille's Law; we will show why it is true in Section 8.4.) A partially clogged artery can be expanded by an operation called angioplasty, in which a balloon-tipped catheter is inflated inside the artery in order to widen it and restore the normal blood flow.

Show that the relative change in F is about four times the relative change in R. How will a 5% increase in the radius affect the flow of blood?

Solution:

$F = kR^4 \quad \Rightarrow \quad dF = 4kR^3\,dR \quad \Rightarrow \quad \dfrac{dF}{F} = \dfrac{4kR^3\,dR}{kR^4} = 4\left(\dfrac{dR}{R}\right)$. Thus, the relative change in F is about 4 times the relative change in R. So a 5% increase in the radius corresponds to a 20% increase in blood flow.

52. Suppose that we don't have a formula for $g(x)$ but we know that $g(2) = -4$ and $g'(x) = \sqrt{x^2+5}$ for all x.

(a) Use a linear approximation to estimate $g(1.95)$ and $g(2.05)$.

(b) Are your estimates in part (a) too large or too small? Explain.

Solution:

(a) $g'(x) = \sqrt{x^2+5} \quad \Rightarrow \quad g'(2) = \sqrt{9} = 3$. $g(1.95) \approx g(2) + g'(2)(1.95 - 2) = -4 + 3(-0.05) = -4.15$.

$g(2.05) \approx g(2) + g'(2)(2.05 - 2) = -4 + 3(0.05) = -3.85$.

(b) The formula $g'(x) = \sqrt{x^2+5}$ shows that $g'(x)$ is positive and increasing. This means that the slopes of the tangent lines are positive and the tangents are getting steeper. So the tangent lines lie *below* the graph of g. Hence, the estimates in part (a) are too small.