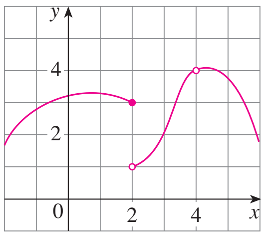
Section 2.2 The Limit of a Function

4. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to 2^-} f(x)$ (b) $\lim_{x \to 2^+} f(x)$ (c) $\lim_{x \to 2} f(x)$ (d) $f(2)$ (e) $\lim_{x \to 4} f(x)$ (f) $f(4)$



Solution:

(a) As x approaches 2 from the left, the values of $f(x)$ approach 3, so $\lim_{x \to 2^-} f(x) = 3$.

(b) As x approaches 2 from the right, the values of $f(x)$ approach 1, so $\lim_{x \to 2^+} f(x) = 1$.

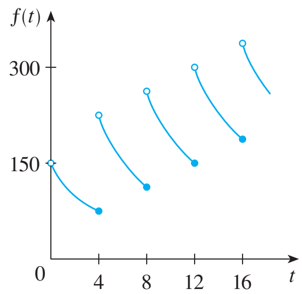
(c) $\lim_{x \to 2} f(x)$ does not exist since the left-hand limit does not equal the right-hand limit.

(d) When $x = 2$, $y = 3$, so $f(2) = 3$.

(e) As x approaches 4, the values of $f(x)$ approach 4, so $\lim_{x \to 4} f(x) = 4$.

(f) There is no value of $f(x)$ when $x = 4$, so $f(4)$ does not exist.

10. A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find $\lim_{t \to 12^-} f(t)$ and $\lim_{t \to 12^+} f(t)$ and explain the significance of these one-sided limits.



Solution:

$\lim_{t \to 12^-} f(t) = 150$ mg and $\lim_{t \to 12^+} f(t) = 300$ mg. These limits show that there is an abrupt change in the amount of drug in the patient's bloodstream at $t = 12$ h. The left-hand limit represents the amount of the drug just before the fourth injection. The right-hand limit represents the amount of the drug just after the fourth injection.

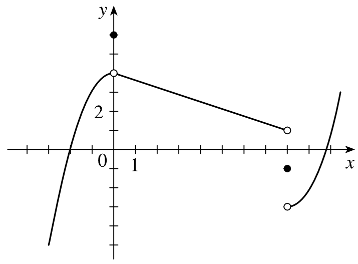
16. Sketch the graph of an example of a function f that satisfies all of the given conditions.

$$\lim_{x\to 0} f(x) = 4,\ \lim_{x\to 8^-} f(x) = 1,\ \lim_{x\to 8^+} f(x) = -3,\ f(0) = 6,\ f(8) = -1$$

Solution:

$\lim_{x\to 0} f(x) = 4,\ \lim_{x\to 8^-} f(x) = 1,\ \lim_{x\to 8^+} f(x) = -3,$

$f(0) = 6,\ f(8) = -1$



38. Determine the infinite limit. $\lim_{x\to 3^-} \frac{x^2+4x}{x^2-2x-3}$

Solution:

$\lim_{x\to 3^-} \frac{x^2+4x}{x^2-2x-3} = \lim_{x\to 3^-} \frac{x^2+4x}{(x-3)(x+1)} = -\infty$ since the numerator is positive and the denominator approaches 0 through negative values as $x \to 3^-$.

42. (a) Find the vertical asymptotes of the function $y = \frac{x^2+1}{3x-2x^2}$

(b) Confirm your answer to part (a) by graphing the function.

Solution:

(a) The denominator of $y = \dfrac{x^2+1}{3x-2x^2} = \dfrac{x^2+1}{x(3-2x)}$ is equal to zero when $x = 0$ and $x = \frac{3}{2}$ (and the numerator is not), so $x = 0$ and $x = 1.5$ are vertical asymptotes of the function.

(b)