Section 2.1 The Tangent and Velocity Problems

3. The point $P(2,-1)$ lies on the curve $y = 1/(1-x)$.

   (a) If Q is the point $(x, 1/(1-x))$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x:

   (i) 1.5 (ii) 1.9 (iii) 1.99 (iv) 1.999 (v) 2.5 (vi) 2.1 (vii) 2.01 (viii) 2.001

   (b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(2,-1)$.

   (c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(2,-1)$.

Solution:

(a) $y = \dfrac{1}{1-x}$, $P(2,-1)$

	x	$Q(x, 1/(1-x))$	m_{PQ}
(i)	1.5	$(1.5, -2)$	2
(ii)	1.9	$(1.9, -1.111\,111)$	1.111 111
(iii)	1.99	$(1.99, -1.010\,101)$	1.010 101
(iv)	1.999	$(1.999, -1.001\,001)$	1.001 001
(v)	2.5	$(2.5, -0.666\,667)$	0.666 667
(vi)	2.1	$(2.1, -0.909\,091)$	0.909 091
(vii)	2.01	$(2.01, -0.990\,099)$	0.990 099
(viii)	2.001	$(2.001, -0.999\,001)$	0.999 001

(b) The slope appears to be 1.

(c) Using $m = 1$, an equation of the tangent line to the curve at $P(2,-1)$ is $y - (-1) = 1(x-2)$, or $y = x - 3$.

6. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t - 1.86t^2$.

   (a) Find the average velocity over the given time intervals: (i) [1,2] (ii) [1,1.5] (iii) [1,1.1] (iv) [1,1.01] (v) [1,1.001]

   (b) Estimate the instantaneous velocity when $t = 1$.

Solution:

(a) $y = y(t) = 10t - 1.86t^2$. At $t = 1$, $y = 10(1) - 1.86(1)^2 = 8.14$. The average velocity between times 1 and $1+h$ is

$$v_{\text{ave}} = \frac{y(1+h) - y(1)}{(1+h) - 1} = \frac{\left[10(1+h) - 1.86(1+h)^2\right] - 8.14}{h} = \frac{6.28h - 1.86h^2}{h} = 6.28 - 1.86h, \text{ if } h \neq 0.$$

(i) $[1,2]$: $h = 1$, $v_{\text{ave}} = 4.42$ m/s

(ii) $[1,1.5]$: $h = 0.5$, $v_{\text{ave}} = 5.35$ m/s

(iii) $[1,1.1]$: $h = 0.1$, $v_{\text{ave}} = 6.094$ m/s

(iv) $[1,1.01]$: $h = 0.01$, $v_{\text{ave}} = 6.2614$ m/s

(v) $[1,1.001]$: $h = 0.001$, $v_{\text{ave}} = 6.27814$ m/s

(b) The instantaneous velocity when $t = 1$ (h approaches 0) is 6.28 m/s.