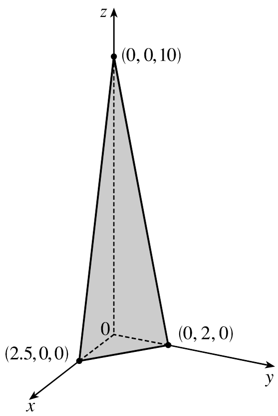
Section 14.1 Functions of Several Variables

25. Sketch the graph of the function.

$$f(x, y) = 10 - 4x - 5y$$

Solution:

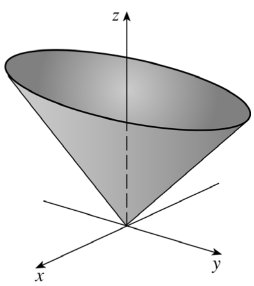
$z = 10 - 4x - 5y$ or $4x + 5y + z = 10$, a plane with intercepts 2.5, 2, and 10.



30. Sketch the graph of the function. $f(x, y) = \sqrt{4x^2 + y^2}$.

Solution:

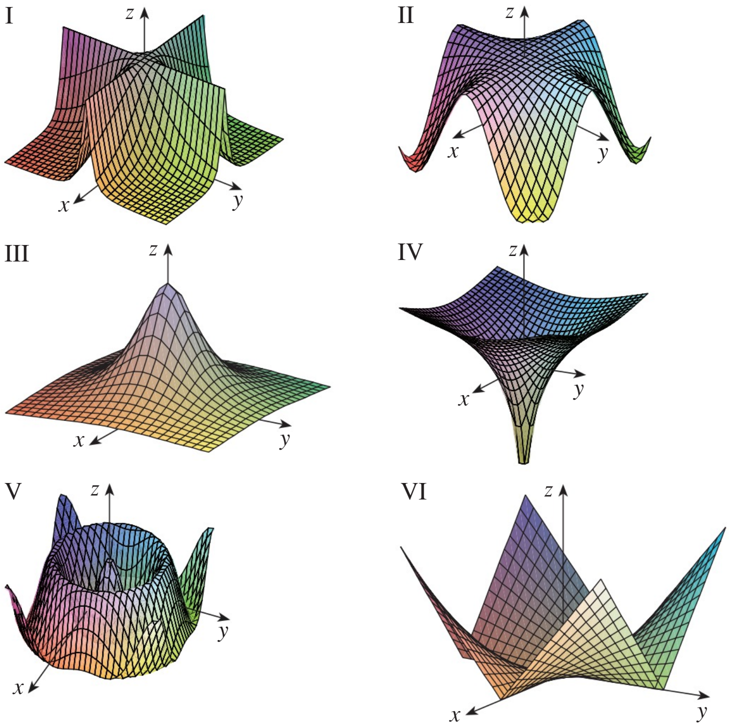
$z = \sqrt{4x^2 + y^2}$ so $4x^2 + y^2 = z^2$ and $z \geq 0$, the top half of an elliptic cone.



32. Match the function with its graph (labeled I–VI). Give reasons for your choices.

(a) $f(x, y) = \frac{1}{1+x^2+y^2}$ (b) $f(x, y) = \frac{1}{1+x^2y^2}$ (c) $f(x, y) = \ln(x^2+y^2)$ (d) $f(x, y) = \cos\sqrt{x^2 + y^2}$ (e) $f(x, y) = |xy|$

(f) $f(x, y) = \cos(xy)$



Solution:

(a) $f(x,y) = \dfrac{1}{1+x^2+y^2}$. The trace in $x=0$ is $z = \dfrac{1}{1+y^2}$, and the trace in $y=0$ is $z = \dfrac{1}{1+x^2}$. The only possibility is graph III. Notice also that the level curves of f are $\dfrac{1}{1+x^2+y^2} = k \quad \Leftrightarrow \quad x^2+y^2 = \dfrac{1}{k} - 1$, a family of circles for $k<1$.

(b) $f(x,y) = \dfrac{1}{1+x^2y^2}$. The trace in $x=0$ is the horizontal line $z=1$, and the trace in $y=0$ is also $z=1$. Both graphs I and II have these traces; however, notice that here $z>0$, so the graph is I.

(c) $f(x,y) = \ln(x^2+y^2)$. The trace in $x=0$ is $z = \ln y^2$, and the trace in $y=0$ is $z=\ln x^2$. The level curves of f are $\ln(x^2+y^2) = k \quad \Leftrightarrow \quad x^2+y^2 = e^k$, a family of circles. In addition, f is large negative when x^2+y^2 is small, so this is graph IV.

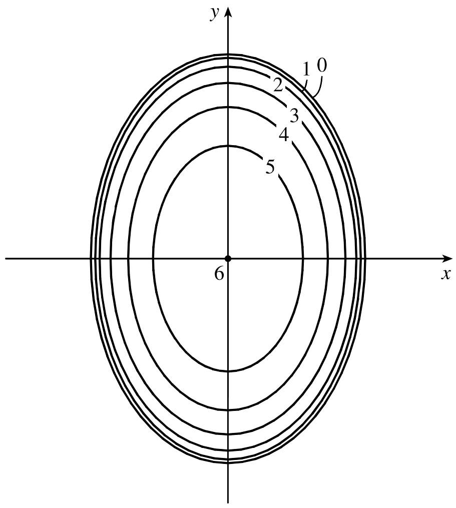
(d) $f(x,y) = \cos\sqrt{x^2+y^2}$. The trace in $x=0$ is $z = \cos\sqrt{y^2} = \cos|y| = \cos y$, and the trace in $y=0$ is $z = \cos\sqrt{x^2} = \cos|x| = \cos x$. Notice also that the level curve $f(x,y)=0$ is $\cos\sqrt{x^2+y^2} = 0 \quad \Leftrightarrow$ $x^2+y^2 = \left(\frac{\pi}{2} + n\pi\right)^2$, a family of circles, so this is graph V.

(e) $f(x,y) = (x^2-y^2)^2$. The trace in $x=0$ is $z=y^4$, and in $y=0$ is $z=x^4$. Notice that the trace in $z=0$ is $0 = (x^2-y^2)^2 \quad \Rightarrow \quad y = \pm x$, so it must be graph VI.

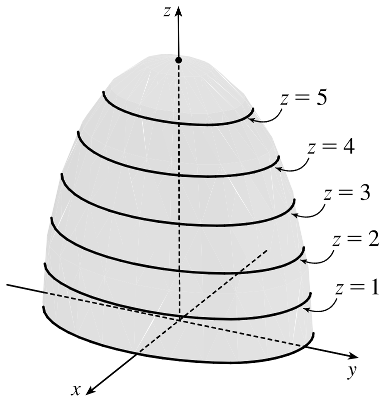
(f) $f(x,y) = \cos(xy)$. The trace in $x=0$ is $z = \cos 0 = 1$, and the trace in $y=0$ is $z=1$. As mentioned in part (b), these traces match both graphs I and II. Here z can be negative, so the graph is II. (Also notice that the trace in $x=1$ is $z=\cos y$, and the trace in $y=1$ is $z=\cos x$.)

54. Sketch both a contour map and a graph of the function and compare them. $f(x,y) = \sqrt{36 - 9x^2 - 4y^2}$.

Solution:



The contour map consists of the level curves $k = \sqrt{36 - 9x^2 - 4y^2} \Rightarrow 9x^2 + 4y^2 = 36 - k^2$, $k \geq 0$, a family of ellipses with major axis the y-axis. (Or, if $k = 6$, the origin.)



The graph of $f(x,y)$ is the surface $z = \sqrt{36 - 9x^2 - 4y^2}$, or equivalently the upper half of the ellipsoid $9x^2 + 4y^2 + z^2 = 36$. If we visualize lifting each ellipse $k = \sqrt{36 - 9x^2 - 4y^2}$ of the contour map to the plane $z = k$, we have horizontal traces that indicate the shape of the graph of f.