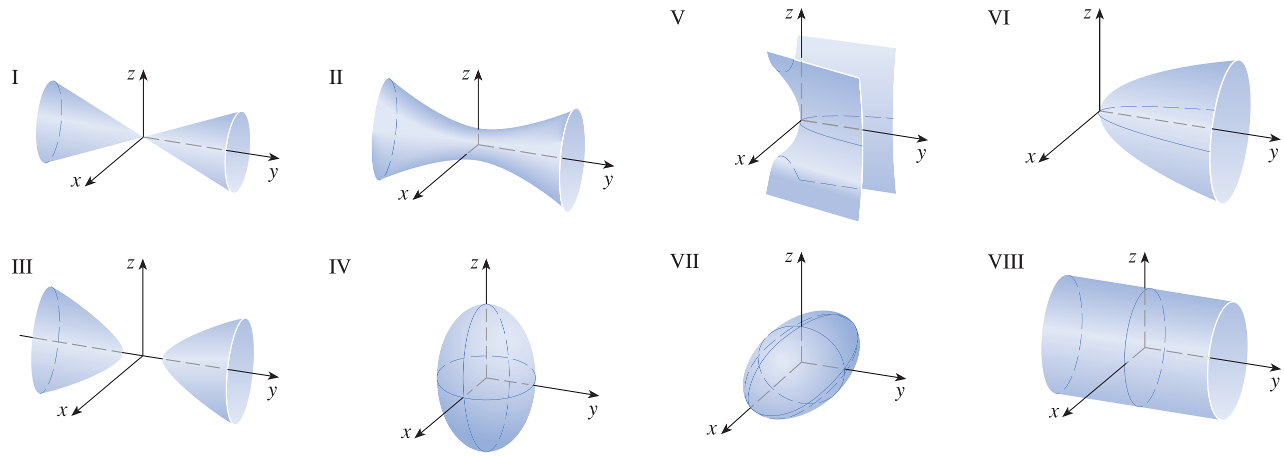
Section 12.6 Cylinders and Quadric Surfaces

26. Match the equation with its graph (labeled I–VIII). Give reasons for your choice. $-x^2 + y^2 - z^2 = 1$

30. Match the equation with its graph (labeled I–VIII). Give reasons for your choice. $y = x^2 - z^2$



Solution:

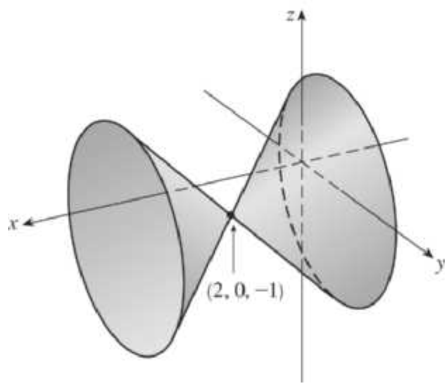
26. $-x^2 + y^2 - z^2 = 1$ is the equation of a hyperboloid of two sheets, with $a = b = c = 1$. This surface does not intersect the xz-plane at all, so the axis of the hyperboloid is the y-axis. Hence, the correct graph is III.

30. $y = x^2 - z^2$ is the equation of a hyperbolic paraboloid. The trace in the xy-plane is the parabola $y = x^2$. So the correct graph is V.

38. Reduce the equation to one of the standard forms, classify the surface, and sketch it. $x^2 - y^2 - z^2 - 4x - 2z + 3 = 0$.

Solution:

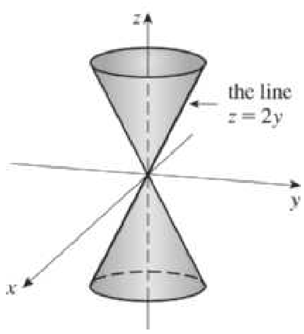
Completing squares in x and z gives $\left(x^2 - 4x + 4\right) - y^2 - \left(z^2 + 2z + 1\right) + 3 = 0 + 4 - 1 \quad \Leftrightarrow$ $(x-2)^2 - y^2 - (z+1)^2 = 0$ or $(x-2)^2 = y^2 + (z+1)^2$, a circular cone with vertex $(2, 0, -1)$ and axis the horizontal line $y = 0$, $z = -1$.



48. Find an equation for the surface obtained by rotating the line $z = 2y$ about the z-axis.

Solution:

Rotating the line $z = 2y$ about the z-axis creates a (right) circular cone with vertex at the origin and axis the z-axis. Traces in $z = k$ ($k \neq 0$) are circles with center $(0, 0, k)$ and radius $y = z/2 = k/2$, so an equation for the trace is $x^2 + y^2 = (k/2)^2$, $z = k$. Thus an equation for the surface is $x^2 + y^2 = (z/2)^2$ or $4x^2 + 4y^2 = z^2$.



50. Find an equation for the surface consisting of all points P for which the distance from P to the x-axis is twice the distance from P to the yz-plane. Identify the surface.

Solution:

Let $P = (x, y, z)$ be an arbitrary point whose distance from the x-axis is twice its distance from the yz-plane. The distance from P to the x-axis is $\sqrt{(x-x)^2 + y^2 + z^2} = \sqrt{y^2 + z^2}$ and the distance from P to the yz-plane ($x = 0$) is $|x|/1 = |x|$. Thus $\sqrt{y^2 + z^2} = 2|x| \quad \Leftrightarrow \quad y^2 + z^2 = 4x^2 \quad \Leftrightarrow \quad x^2 = (y^2/2^2) + (z^2/2^2)$. So the surface is a right circular cone with vertex the origin and axis the x-axis.