Section 1.5 Inverse Functions and Logarithms

43. Use the laws of logarithms to expand each expression. (a) $\log_{10}(x^2y^3z)$ (b) $\ln\left(\frac{x^4}{\sqrt{x^2-4}}\right)$

Solution:

(a) $\log_{10}\left(x^2y^3z\right) = \log_{10} x^2 + \log_{10} y^3 + \log_{10} z$ [Law 1]

$= 2\log_{10} x + 3\log_{10} y + \log_{10} z$ [Law 3]

(b) $\ln\left(\frac{x^4}{\sqrt{x^2-4}}\right) = \ln x^4 - \ln(x^2-4)^{1/2}$ [Law 2]

$= 4\ln x - \frac{1}{2}\ln[(x+2)(x-2)]$ [Law 3]

$= 4\ln x - \frac{1}{2}[\ln(x+2) + \ln(x-2)]$ [Law 1]

$= 4\ln x - \frac{1}{2}\ln(x+2) - \frac{1}{2}\ln(x-2)$

56. $f(x) = \ln(x-1) - 1$

(a) What are the domain and range of f?

(b) What is the x-intercept of the graph of f?

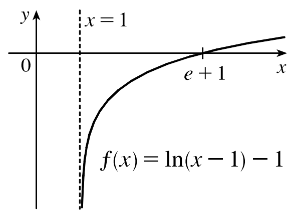
(c) Sketch the graph of f.

Solution:

(a) The domain of $f(x) = \ln(x-1) - 1$ is $x > 1$ and the range is $\mathbb{R}$.

(b) $y = 0 \Rightarrow \ln(x-1) - 1 = 0 \Rightarrow \ln(x-1) = 1 \Rightarrow$
$x - 1 = e^1 \Rightarrow x = e + 1$

(c) We shift the graph of $y = \ln x$ one unit to the right and one unit downward.



74. Find the exact value of each expression. (a) $\arcsin(\sin(5\pi/4))$ (b) $\cos(2\sin^{-1}(\frac{5}{13}))$

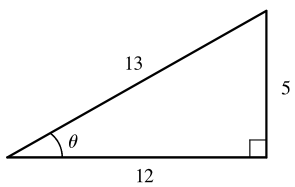
Solution:

(a) $\arcsin(\sin(5\pi/4)) = \arcsin\left(-1/\sqrt{2}\right) = -\frac{\pi}{4}$ because $\sin\left(-\frac{\pi}{4}\right) = -1/\sqrt{2}$ and $-\frac{\pi}{4}$ is in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(b) Let $\theta = \sin^{-1}\left(\frac{5}{13}\right)$ [see the figure].

$\cos\left(2\sin^{-1}\left(\frac{5}{13}\right)\right) = \cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$

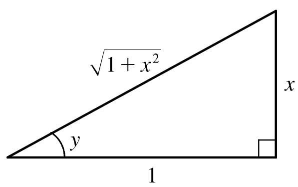


77. Simplify the expression. $\sin(\tan^{-1} x)$.

Solution:

Let $y = \tan^{-1} x$. Then $\tan y = x$, so from the triangle (which illustrates the case $y > 0$), we see that

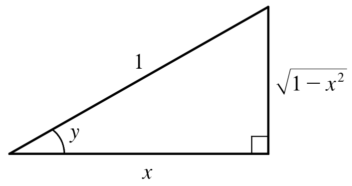
$\sin(\tan^{-1} x) = \sin y = \dfrac{x}{\sqrt{1+x^2}}$.



78. Simplify the expression. $\sin(2\arccos x)$.

Solution:

Let $y = \arccos x$. Then $\cos y = x$, so from the triangle (which illustrates the case $y > 0$), we see that



$$\begin{aligned} \sin(2\arccos x) &= \sin 2y = 2\sin y\, \cos y \\ &= 2(\sqrt{1-x^2}\,)(x) = 2x\sqrt{1-x^2} \end{aligned}$$