Section 5.4 Indefinite Integrals and the Net Change Theorem

22. Find the general indefinite integral. $\int \sec t(\sec t + \tan t)dt$

    Solution:

    $\int \sec t\,(\sec t + \tan t)\,dt = \int (\sec^2 t + \sec t\,\tan t)\,dt = \tan t + \sec t + C$

54. Evaluate the definite integral. $\int_0^{\frac{3\pi}{2}} |\sin x| dx$.

    Solution:

    $\int_0^{3\pi/2} |\sin x|\; dx = \int_0^{\pi} \sin x\,dx + \int_{\pi}^{3\pi/2} (-\sin x)\,dx = \big[-\cos x\big]_0^{\pi} + \big[\cos x\big]_{\pi}^{3\pi/2} = [1-(-1)] + [0-(-1)] = 2+1 = 3$

72. The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find (a) the velocity at time t and (b) the distance traveled during the given time interval.

    $$a(t) = 2t+3,\ \ v(0) = -4,\ \ 0 \le t \le 3.$$

    Solution:

    (a) $v'(t) = a(t) = 2t+3 \quad\Rightarrow\quad v(t) = t^2 + 3t + C \quad\Rightarrow\quad v(0) = C = -4 \quad\Rightarrow\quad v(t) = t^2 + 3t - 4$

    (b) Distance traveled $= \int_0^3 \left|t^2 + 3t - 4\right| dt = \int_0^3 |(t+4)(t-1)|\,dt = \int_0^1 \left(-t^2 - 3t + 4\right) dt + \int_1^3 \left(t^2 + 3t - 4\right) dt$

    $= \left[-\frac{1}{3}t^3 - \frac{3}{2}t^2 + 4t\right]_0^1 + \left[\frac{1}{3}t^3 + \frac{3}{2}t^2 - 4t\right]_1^3$

    $= \left(-\frac{1}{3} - \frac{3}{2} + 4\right) + \left(9 + \frac{27}{2} - 12\right) - \left(\frac{1}{3} + \frac{3}{2} - 4\right) = \frac{89}{6}$ m

77. The marginal cost of manufacturing x yards of a certain fabric is

    $$C'(x) = 3 - 0.01x + 0.000006x^2$$

    (in dollars per yard). Find the increase in cost if the production level is raised from 2000 yards to 4000 yards.

    Solution:

    From the Net Change Theorem, the increase in cost if the production level is raised from 2000 m to 4000 m is

    $C(4000) - C(2000) = \int_{2000}^{4000} C'(x)\,dx$.

    $$\int_{2000}^{4000} C'(x)\,dx = \int_{2000}^{4000} (3 - 0.01x + 0.000\,006x^2)\,dx = \Big[3x - 0.005x^2 + 0.000\,002x^3\Big]_{2000}^{4000}$$

    $$= 60\,000 - 2000 = \$58\,000$$