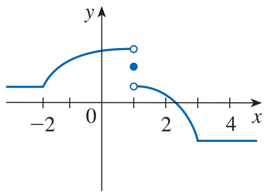
Section 2.8 The Derivative as a Function

40. Suppose N is the number of people in the United States who travel by car to another state for a vacation this year when the average price of gasoline is p dollars per gallon. Do you expect dN/dp to be positive or negative? Explain.

Solution:

dN/dp is the rate at which the number of people who travel by car to another province for a vacation changes with respect to the price of gasoline. If the price of gasoline goes up, we would expect fewer people to travel, so we would expect dN/dp to be negative.

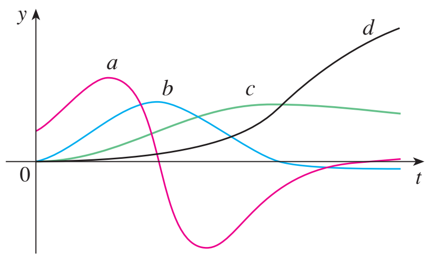
44. The graph of f is given. State, with reasons, the numbers at which f is not differentiable.



Solution:

f is not differentiable at $x = -2$ and $x = 3$, because the graph has corners there, and at $x = 1$, because there is a discontinuity there.

52. The figure shows the graphs of four functions. One is the position function of a car, one is the velocity of the car, one is its acceleration, and one is its jerk. Identify each curve, and explain your choices.



Solution:

a must be the jerk since none of the graphs are 0 at its high and low points. a is 0 where b has a maximum, so $b' = a$. b is 0 where c has a maximum, so $c' = b$. We conclude that d is the position function, c is the velocity, b is the acceleration, and a is the jerk.

58. (a) If $g(x) = x^{2/3}$, show that $g'(0)$ does not exist.

(b) If $a \neq 0$, find $g'(a)$.

(c) Show that $y = x^{2/3}$ has a vertical tangent line at $(0, 0)$.

(d) Illustrate part (c) by graphing $y = x^{2/3}$.

Solution:

(a) $g'(0) = \lim\limits_{x\to 0} \dfrac{g(x) - g(0)}{x - 0} = \lim\limits_{x\to 0} \dfrac{x^{2/3} - 0}{x} = \lim\limits_{x\to 0} \dfrac{1}{x^{1/3}}$, which does not exist.

(b) $g'(a) = \lim\limits_{x\to a} \dfrac{g(x) - g(a)}{x - a} = \lim\limits_{x\to a} \dfrac{x^{2/3} - a^{2/3}}{x - a} = \lim\limits_{x\to a} \dfrac{(x^{1/3} - a^{1/3})(x^{1/3} + a^{1/3})}{(x^{1/3} - a^{1/3})(x^{2/3} + x^{1/3}a^{1/3} + a^{2/3})}$

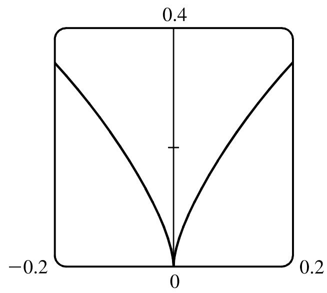
$$= \lim_{x\to a} \frac{x^{1/3} + a^{1/3}}{x^{2/3} + x^{1/3}a^{1/3} + a^{2/3}} = \frac{2a^{1/3}}{3a^{2/3}} = \frac{2}{3a^{1/3}} \text{ or } \tfrac{2}{3}a^{-1/3}$$

(c) $g(x) = x^{2/3}$ is continuous at $x = 0$ and

$\lim\limits_{x\to 0} |g'(x)| = \lim\limits_{x\to 0} \dfrac{2}{3\,|x|^{1/3}} = \infty$. This shows that

g has a vertical tangent line at $x = 0$.

(d)



63. **Derivatives of Even and Odd Function** Recall that a function f is called *even* if $f(-x) = f(x)$ for all x in its domain and *odd* if $f(-x) = -f(x)$ for all such x. Prove each of the following.

(a) The derivative of an even function is an odd function.

(b) The derivative of an odd function is an even function.

Solution:

(a) If f is even, then

$$\begin{aligned} f'(-x) &= \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h\to 0} \frac{f[-(x-h)] - f(-x)}{h} \\ &= \lim_{h\to 0} \frac{f(x-h) - f(x)}{h} = -\lim_{h\to 0} \frac{f(x-h) - f(x)}{-h} \qquad [\text{let } \Delta x = -h] \\ &= -\lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = -f'(x) \end{aligned}$$

Therefore, f' is odd.

(b) If f is odd, then

$$\begin{aligned} f'(-x) &= \lim_{h\to 0} \frac{f(-x+h) - f(-x)}{h} = \lim_{h\to 0} \frac{f[-(x-h)] - f(-x)}{h} \\ &= \lim_{h\to 0} \frac{-f(x-h) + f(x)}{h} = \lim_{h\to 0} \frac{f(x-h) - f(x)}{-h} \qquad [\text{let } \Delta x = -h] \\ &= \lim_{\Delta x\to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = f'(x) \end{aligned}$$

Therefore, f' is even.