Section 2.6 Limits at Infinity; Horizontal Asymptotes

30. Find the limit or show that it does not exist.

$$\lim_{x\to-\infty}\left(\sqrt{4x^2+3x}+2x\right).$$

Solution:

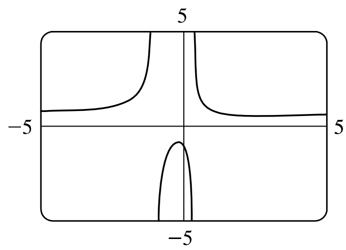
$$\begin{aligned}
\lim_{x\to-\infty}\left(\sqrt{4x^2+3x}+2x\right) &= \lim_{x\to-\infty}\left(\sqrt{4x^2+3x}+2x\right)\left[\frac{\sqrt{4x^2+3x}-2x}{\sqrt{4x^2+3x}-2x}\right]\\
&= \lim_{x\to-\infty}\frac{\left(4x^2+3x\right)-(2x)^2}{\sqrt{4x^2+3x}-2x} = \lim_{x\to-\infty}\frac{3x}{\sqrt{4x^2+3x}-2x}\\
&= \lim_{x\to-\infty}\frac{3x/x}{\left(\sqrt{4x^2+3x}-2x\right)/x} = \lim_{x\to-\infty}\frac{3}{-\sqrt{4+3/x}-2} \qquad \text{[since } x=-\sqrt{x^2} \text{ for } x<0\text{]}\\
&= \frac{3}{-\sqrt{4+0}-2} = -\frac{3}{4}
\end{aligned}$$

48. Find the horizontal and vertical asymptotes of each curve. If you have a graphing device, check your work by graphing the curve and estimating the asymptotes.

$$y=\frac{2x^2+1}{3x^2+2x-1}.$$

Solution:

$$\begin{aligned}
\lim_{x\to\pm\infty}\frac{2x^2+1}{3x^2+2x-1} &= \lim_{x\to\pm\infty}\frac{(2x^2+1)/x^2}{(3x^2+2x-1)/x^2}\\
&= \lim_{x\to\pm\infty}\frac{2+1/x^2}{3+2/x-1/x^2} = \frac{2}{3}
\end{aligned}$$



so $y=\frac{2}{3}$ is a horizontal asymptote. $y=f(x)=\frac{2x^2+1}{3x^2+2x-1}=\frac{2x^2+1}{(3x-1)(x+1)}$.

The denominator is zero when $x=\frac{1}{3}$ and -1, but the numerator is nonzero, so $x=\frac{1}{3}$ and $x=-1$ are vertical asymptotes. The graph confirms our work.

58. Find a formula for a function that has vertical asymptotes $x=1$ and $x=3$ and horizontal asymptote $y=1$.

Solution:

Since the function has vertical asymptotes $x=1$ and $x=3$, the denominator of the rational function we are looking for must have factors $(x-1)$ and $(x-3)$. Because the horizontal asymptote is $y=1$, the degree of the numerator must equal the degree of the denominator, and the ratio of the leading coefficients must be 1. One possibility is $f(x)=\frac{x^2}{(x-1)(x-3)}$.