Section 15.1 Double Integrals over Rectangles

22. Calculate the iterated integral. $\int_0^1 \int_0^2 ye^{x-y} dx dy$

Solution:

$$\begin{aligned}\int_0^1 \int_0^2 ye^{x-y}\,dx\,dy &= \int_0^1 \int_0^2 ye^{x}e^{-y}\,dx\,dy = \int_0^2 e^x\,dx \int_0^1 ye^{-y}\,dy \quad \text{[by Equation 11]}\\ &= [e^x]_0^2 \left[(-y-1)e^{-y}\right]_0^1 \quad \text{[by integrating by parts]}\\ &= \left(e^2 - e^0\right)\left[-2e^{-1} - \left(-e^0\right)\right] = (e^2-1)(1-2e^{-1}) \text{ or } e^2 - 2e + 2e^{-1} - 1\end{aligned}$$

34. Calculate the double integral. $\iint_R \frac{1}{1+x+y} dA$, $R = [1,3] \times [1,2]$.

Solution:

$$\begin{aligned}\iint_R \frac{1}{1+x+y}\,dA &= \int_1^3 \int_1^2 \frac{1}{1+x+y}\,dy\,dx = \int_1^3 [\ln(1+x+y)]_{y=1}^{y=2}\,dx = \int_1^3 [\ln(x+3) - \ln(x+2)]\,dx\\ &= \Big[\big((x+3)\ln(x+3) - (x+3)\big) - \big((x+2)\ln(x+2) - (x+2)\big)\Big]_1^3\\ &\qquad \text{[by integrating by parts separately for each term]}\\ &= (6\ln 6 - 6 - 5\ln 5 + 5) - (4\ln 4 - 4 - 3\ln 3 + 3) = 6\ln 6 - 5\ln 5 - 4\ln 4 + 3\ln 3\end{aligned}$$

54. Find the average value of f over the given rectangle.

$$f(x,y) = e^y\sqrt{x+e^y}, \quad R = [0,4]\times[0,1]$$

Solution:

$A(R) = 4 \cdot 1 = 4$, so

$$\begin{aligned}f_{\text{ave}} &= \frac{1}{A(R)} \iint_R f(x,y)\,dA = \tfrac{1}{4}\int_0^4 \int_0^1 e^y\sqrt{x+e^y}\,dy\,dx = \tfrac{1}{4}\int_0^4 \left[\tfrac{2}{3}(x+e^y)^{3/2}\right]_{y=0}^{y=1} dx\\ &= \tfrac{1}{4}\cdot\tfrac{2}{3}\int_0^4 [(x+e)^{3/2} - (x+1)^{3/2}]\,dx = \tfrac{1}{6}\left[\tfrac{2}{5}(x+e)^{5/2} - \tfrac{2}{5}(x+1)^{5/2}\right]_0^4\\ &= \tfrac{1}{6}\cdot\tfrac{2}{5}[(4+e)^{5/2} - 5^{5/2} - e^{5/2} + 1] = \tfrac{1}{15}[(4+e)^{5/2} - e^{5/2} - 5^{5/2} + 1] \approx 3.327\end{aligned}$$