Section 1.4 Exponential Functions

2. Use the Laws of Exponents to rewrite and simplify each expression.

(a) $\frac{\sqrt[3]{4}}{\sqrt[3]{108}}$ (b) $27^{2/3}$ (c) $2x^2(3x^5)^2$ (d) $(2x^{-2})^{-3}x^{-3}$ (e) $\frac{3a^{3/2}\cdot a^{1/2}}{a^{-1}}$ (f) $\frac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}}$

Solution:

(a) $\dfrac{\sqrt[3]{4}}{\sqrt[3]{108}} = \dfrac{\sqrt[3]{4}}{\sqrt[3]{4\cdot 27}} = \dfrac{\sqrt[3]{4}}{\sqrt[3]{4}\cdot\sqrt[3]{27}} = \dfrac{1}{\sqrt[3]{27}} = \dfrac{1}{3}$

(b) $27^{2/3} = (27^{1/3})^2 = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$

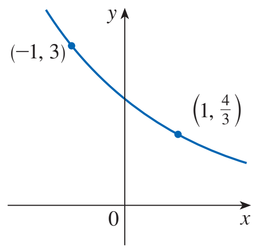
(c) $2x^2(3x^5)^2 = 2x^2\cdot 3^2(x^5)^2 = 2x^2\cdot 9x^{10} = 2\cdot 9x^{2+10} = 18x^{12}$

(d) $(2x^{-2})^{-3}x^{-3} = 2^{-3}(x^{-2})^{-3}x^{-3} = \dfrac{x^6\cdot x^{-3}}{2^3} = \dfrac{x^{6+(-3)}}{8} = \dfrac{x^3}{8}$

(e) $\dfrac{3a^{3/2}\cdot a^{1/2}}{a^{-1}} = 3a^{3/2+1/2}\cdot a^1 = 3a^2\cdot a = 3a^3$

(f) $\dfrac{\sqrt{a\sqrt{b}}}{\sqrt[3]{ab}} = \dfrac{(ab^{1/2})^{1/2}}{(ab)^{1/3}} = \dfrac{a^{1/2}\,(b^{1/2})^{1/2}}{a^{1/3}\,b^{1/3}} = \dfrac{a^{1/2}\,b^{1/4}}{a^{1/3}\,b^{1/3}} = a^{1/2-1/3}\,b^{1/4-1/3} = a^{1/6}\,b^{-1/12} = \dfrac{a^{1/6}}{b^{1/12}} = \dfrac{\sqrt[6]{a}}{\sqrt[12]{b}}$

20. Find the exponential function $f(x) = Cb^x$ whose graph is given.



Solution:

Use $y = Cb^x$ with the points $(-1,3)$ and $\left(1,\frac{4}{3}\right)$. From the point $(-1,3)$, we have $3 = Cb^{-1}$, hence $C = 3b$. Using this and the point $\left(1,\frac{4}{3}\right)$, we get $\frac{4}{3} = Cb^1 \;\Rightarrow\; \frac{4}{3} = (3b)b \;\Rightarrow\; \frac{4}{9} = b^2 \;\Rightarrow\; b = \frac{2}{3}$ [since $b > 0$] and $C = 3(\frac{2}{3}) = 2$. The function is $f(x) = 2(\frac{2}{3})^x$.