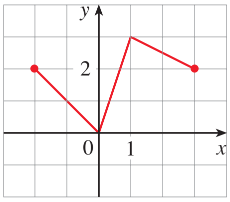
Section 1.3 New Functions from Old Functions

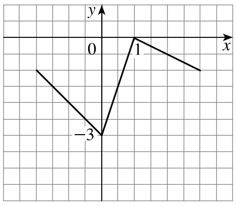
4. The graph of f is given. Draw the graphs of the following functions.

   (a) $y = f(x) - 3$ (b) $y = f(x+1)$ (c) $y = \frac{1}{2}f(x)$ (d) $y = -f(x)$

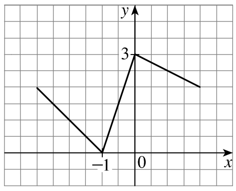


Solution:

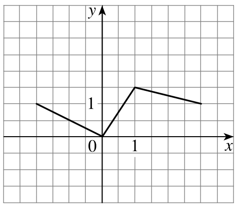
(a) $y = f(x) - 3$: Shift the graph of f 3 units down.



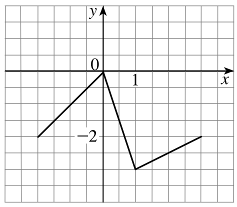
(b) $y = f(x+1)$: Shift the graph of f 1 unit to the left.



(c) $y = \frac{1}{2}f(x)$: Shrink the graph of f vertically by a factor of 2.



(d) $y = -f(x)$: Reflect the graph of f about the x-axis.



36. Find the functions (a) $f \circ g$, (b) $g \circ f$, (c) $f \circ f$, and (d) $g \circ g$ and their domains.

$$f(x) = \frac{1}{x}, g(x) = 2x + 1$$

Solution:

$f(x) = 1/x$ and $g(x) = 2x + 1$. The domain of f is $(-\infty, 0) \cup (0, \infty)$. The domain of g is $(-\infty, \infty)$.

(a) $(f \circ g)(x) = f(g(x)) = f(2x+1) = \dfrac{1}{2x+1}$. The domain is $\{x \mid 2x + 1 \neq 0\} = \{x \mid x \neq -\frac{1}{2}\} = \left(-\infty, -\frac{1}{2}\right) \cup \left(-\frac{1}{2}, \infty\right)$.

(b) $(g \circ f)(x) = g(f(x)) = g\left(\dfrac{1}{x}\right) = 2\left(\dfrac{1}{x}\right) + 1 = \dfrac{2}{x} + 1$. We must have $x \neq 0$, so the domain is $(-\infty, 0) \cup (0, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = x$. Since f requires $x \neq 0$, the domain is $(-\infty, 0) \cup (0, \infty)$.

(d) $(g \circ g)(x) = g(g(x)) = g(2x+1) = 2(2x+1) + 1 = 4x + 3$. The domain is $(-\infty, \infty)$.

54. Express the function in the form $H(t) = \cos(\sqrt{\tan t} + 1)$

Solution:

Let $h(t) = \tan t$, $g(t) = \sqrt{t} + 1$, and $f(t) = \cos t$. Then

$(f \circ g \circ h)(t) = f\left(g(h(t))\right) = f\left(g(\tan t)\right) = f\left(\sqrt{\tan t} + 1\right) = \cos\left(\sqrt{\tan t} + 1\right) = H(t)$.