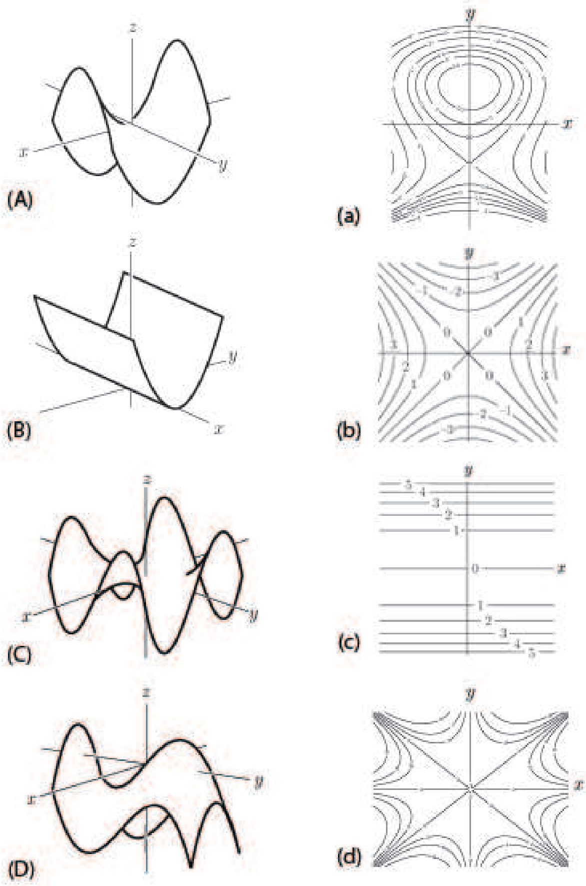
微乙小考一 (2020/3/19)

1. (4 pts) 配合題：將下圖左邊函數圖形與右邊對應的等高線圖匹配。

(A)	(B)	(C)	(D)
b	c	d	a



2. (6 pts) 求 $z = \tan^{-1}\frac{y}{x}$ 在 $x = 2$, $y = -2$ 時之切面方程式。

sol:

$$\frac{\partial z}{\partial x} = \frac{1}{1+\frac{y^2}{x^2}} \cdot (\frac{y}{x})' = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{-y}{x^2} = \frac{-y}{x^2+y^2} \Rightarrow \frac{\partial z}{\partial x}(2,-2) = \frac{1}{4}$$

$$\frac{\partial z}{\partial y} = \frac{1}{1+\frac{y^2}{x^2}} \cdot \frac{1}{x} = \frac{x}{x^2+y^2} \Rightarrow \frac{\partial z}{\partial y}(2,-2) = \frac{1}{4}$$

$$z(2,-2) = tan^{-1}-1 = \frac{-\pi}{4}$$

$$\therefore tangent-plane : z = \frac{-\pi}{4} + \frac{1}{4}(x-2) + \frac{1}{4}(y+2) = \frac{1}{4}(x+y-\pi)$$

3. (6 pts) 已知三角形餘弦定律為：$c^2 = a^2 + b^2 - 2ab\cos\theta$.

在$a = 4$, $b = 3$, $\theta = \frac{\pi}{2}$時，以線性逼近用 Δa, Δb, $\Delta\theta$ 表示 Δc.

sol: by law of cosine: $c^2 = a^2 + b^2 - 2abcos\theta$ or $c = \sqrt{a^2 + b^2 - 2abcos\theta}, c = c(a, b, \theta)$

$$\begin{aligned}
&\Delta c = \frac{\partial c}{\partial a}\Delta a + \frac{\partial c}{\partial b}\Delta b + \frac{\partial c}{\partial \theta}\Delta\theta, where \\
&\frac{\partial c}{\partial a} = \frac{1}{2} \cdot \frac{1}{\sqrt{a^2 + b^2 - 2abcos\theta}} \cdot (2a - 2bcos\theta) = \frac{2a - 2bcos\theta}{2c} \Rightarrow \frac{\partial c}{\partial a}(4, 3, \frac{\pi}{2}) = \frac{4}{5} \\
&\frac{\partial c}{\partial b} = \frac{1}{2} \cdot \frac{1}{\sqrt{a^2 + b^2 - 2abcos\theta}} \cdot (2b - 2acos\theta) = \frac{2b - 2acos\theta}{2c} \Rightarrow \frac{\partial c}{\partial b}(4, 3, \frac{\pi}{2}) = \frac{3}{5} \\
&\frac{\partial c}{\partial \theta} = \frac{1}{2} \cdot \frac{1}{\sqrt{a^2 + b^2 - 2abcos\theta}} \cdot (2absin\theta) = \frac{2absin\theta}{2c} \Rightarrow \frac{\partial c}{\partial \theta}(4, 3, \frac{\pi}{2}) = \frac{12}{5} \\
&\therefore \Delta c = \frac{4\Delta a + 3\Delta b + 12\Delta\theta}{5}
\end{aligned}$$

4. (4 pts) 填充題：$f(x, y)$ 為一雙變數函數, 且 $x(u, v) = u - v$, $y(u, v) = u + v$ 令 $g(u, v) = f(x(u, v), y(u, v))$ 為合成函數. 根據下表資料填答.

(x, y)	$(0, 0)$	$(1, 0)$	$(0, 1)$	$(2, 0)$	$(1, 1)$	$(0, 2)$
$\frac{\partial f}{\partial x}$	π	π^2	π^3	π^4	π^5	π^6
$\frac{\partial f}{\partial y}$	1	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{5}$	$\sqrt{7}$	$\sqrt{11}$

$\frac{\partial g}{\partial u}(1, 1)$ = ____________________ ; $\frac{\partial g}{\partial v}(1, 1)$ = ____________________.

sol:

$$\begin{aligned}
\frac{\partial g}{\partial u}(1, 1) &= \frac{\partial g}{\partial x}(2, 0)\frac{dx}{du} + \frac{\partial g}{\partial y}(2, 0)\frac{dy}{du} = \pi^6 + \sqrt{11} \\
\frac{\partial g}{\partial v}(1, 1) &= \frac{\partial g}{\partial x}(2, 0)\frac{dx}{dv} + \frac{\partial g}{\partial y}(2, 0)\frac{dy}{dv} = -\pi^6 + \sqrt{11}
\end{aligned}$$