

微乙小考五 (2019/12/12)

1. (8 pts) 計算以下的積分。 (a) (4 pts) $\int 4x \ln(2x + 1) dx$ (b) (4 pts) $\int \sin^{-1} x dx$

sol: (Integral by part)

(a)

$$\begin{aligned} \int 4x \ln(2x + 1) dx &= 2x^2 \ln(2x + 1) - 4 \int \frac{x^2}{2x + 1} dx \\ &= 2x^2 \ln(2x + 1) - 4 \int \left(\frac{x}{2} - \frac{1}{4} + \frac{\frac{1}{4}}{2x + 1} \right) dx \\ &= 2x^2 \ln(2x + 1) - x^2 + x - \frac{1}{2} \ln(2x + 1) + C \end{aligned}$$

◇ Let $u = \ln(2x + 1)$ and $dv = 4x dx$ □

(b)

$$\int \sin^{-1} x dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx = x \sin^{-1} x + \sqrt{1-x^2} + C$$

◇ Let $u = \sin^{-1} x$ and $dv = 1 dx$ □

2. (6 pts) 計算以下的積分。 (a) (3 pts) $\int \frac{1}{x^2 - 6x + 10} dx$ (b) (3 pts) $\int \sqrt{\tan x} \sec^2 x dx$

sol: (Change variables)

(a)

$$\begin{aligned} \int \frac{1}{x^2 - 6x + 10} dx &= \int \frac{1}{(x-3)^2 + 1} dx \quad (\text{Let } x-3 = \tan\theta, dx = \sec^2\theta d\theta) \\ &= \int \frac{\sec^2\theta}{\sec^2\theta} d\theta \\ &= \theta + C = \arctan(x-3) + C \end{aligned}$$

□

(b)

$$\begin{aligned} \int \sqrt{\tan x} \sec^2 x dx &= \int \sqrt{u} du \quad (\text{Let } u = \tan x, du = \sec^2 x dx) \\ &= \frac{2}{3} u^{\frac{3}{2}} + C \\ &= \frac{2}{3} (\tan x)^{\frac{3}{2}} + C \end{aligned}$$

□

3. (6 pts)

(a) (3 pts) 把 $\frac{4x^2 + 4x + 7}{(x+2)(x^2+1)}$ 寫成部分分式和。

(b) (3 pts) 計算 $\int \frac{4x^2 + 4x + 7}{(x+2)(x^2+1)} dx$ 。

sol: (a) $\frac{4x^2 + 4x + 7}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1} = \frac{(a+b)x^2 + (2b+c)x + (a+2c)}{(x+2)(x^2+1)}$

$$\Rightarrow \begin{cases} a=3 \\ b=1 \\ c=2 \end{cases} \Rightarrow \frac{4x^2 + 4x + 7}{(x+2)(x^2+1)} = \frac{3}{x+2} + \frac{x+2}{x^2+1}$$

□

(b)

$$\begin{aligned} \int \frac{4x^2 + 4x + 7}{(x+2)(x^2+1)} dx &= \int \left(\frac{3}{x+2} + \frac{x+2}{x^2+1} \right) dx \\ &= \int \left(\frac{3}{x+2} + \frac{x}{x^2+1} + \frac{2}{1+x^2} \right) dx \\ &= 3 \ln|x+2| + \frac{1}{2} \ln(x^2+1) + 2 \arctan(x) + C \end{aligned}$$

□