微乙小考四 (2018/5/24)

1. (10分) 解微分方程 $y'(t) = y^2(t)$.

sol: Since this differential equation is seperable.

$$\int \frac{1}{y^2} dy = \int 1 \cdot dt$$
$$\frac{-1}{y} = t + C$$
$$y = \frac{-1}{t+C}$$

where C is a constant.

The trivial solution is $y(t) = 0$.

2. (10分) 解微分方程 $\begin{cases} t\dfrac{dy}{dt} + y = \sin t \quad t > 0, \\ y(\pi/2) = 1. \end{cases}$.

sol: For $t\dfrac{dy}{dt} + y = \sin t$, divide both side by t we get

$$\frac{dy}{dt} + \frac{y}{t} = \frac{\sin t}{t}$$

Then integrating factor is

$$u(t) = e^{\int \frac{dt}{t}} = e^{\ln t} = t$$

With $u(t)$ we can solve first order ODE now

$$t\frac{dy}{dt} + y = (ty)' = \sin t$$
$$ty = \int \sin t dt = -\cos t + C$$
$$y = \frac{-\cos t + C}{t}$$

where C is a constant.

Since we are given the initial condition, we can find the unique solution of ODE by determined the value of C

$$1 = \frac{0 + C}{\frac{\pi}{2}}$$

which imply $C = \dfrac{\pi}{2}$ Hence the unique solution is

$$y = \frac{-\cos t + \frac{\pi}{2}}{t}$$