微乙小考三 (2018/4/19)

1. (6分) 求二重積分

$$\iint_{\Omega} x^2(x^2+y^3)+ye^{xy}\,dA \qquad \Omega=[1,2]\times[-1,1].$$

sol:

$$\int_{-1}^{1}\int_{1}^{2} x^2(x^2+y^3)+ye^{xy}dxdy$$

$$=\int_{-1}^{1}\frac{x^5}{5}+\frac{x^3y^3}{3}+e^{xy}\ \Big|_1^2\ dy$$

$$=\int_{-1}^{1}\frac{31}{5}+\frac{7y^3}{3}+e^{2y}-e^ydy=\frac{31y}{5}+\frac{7y^4}{12}+\frac{e^{2y}}{2}-e^y\ \Big|_{-1}^{1}$$

$$=\frac{62}{5}+\frac{e^2}{2}-\frac{1}{2e^2}-e+\frac{1}{e}$$

– score: First and second integral get 3 points and wrong answer lose one point.

2. (6分) 求二重積分

$$\iint_{\Omega} xy^2\,dA, \qquad \Omega=\{(x,y): x^2\le y \text{ and } y^2\le x\}.$$

sol:

$$\begin{cases} x^2-y=0\to x^2=y \\ y^2-x=0\to \text{from upper equ}\to x^4-x=0 \end{cases} \tag{1}$$

$\to x(x^3-1)=0$,then $x=0,1$ and $y=0,1$

(sol 1.)

$$\int_0^1\int_{y^2}^{\sqrt{y}} xy^2dxdy$$

$$=\int_0^1\frac{x^2y^2}{2}\ \Big|_{y^2}^{\sqrt{y}}dy$$

$$=\int_0^1\frac{1}{2}(y^3-y^6)dy=\frac{y^4}{8}-\frac{y^7}{14}\Big|_0^1=\frac{3}{56}$$

(sol 2.)

$$\int_0^1\int_{x^2}^{\sqrt{x}} xy^2dydx$$

$$=\int_0^1\frac{xy^3}{3}\Big|_{x^2}^{\sqrt{x}}dx=\int_0^1\frac{1}{3}(x^{\frac{5}{2}}-x^7)dx=\frac{1}{3}(\frac{2}{7}x^3-\frac{1}{8}x^8)\Big|_0^1=\frac{3}{56}$$

– score: Draw picture to find the interval get 3 points. Right interval of integral gets 3 points. Wrong answer lose 1 point.

3. (8分) 在限制條件 $x^2+xy+y^2=\frac{13}{4}$ 下，求 $f(x,y)=(x-1)(y-1)$ 的所有的極點，並找出最大值及最小值。

sol:

$$\begin{cases} \frac{\partial f}{\partial x}=\lambda\frac{\partial x^2+xy+y^2-\frac{13}{4}}{\partial x} \\ \frac{\partial f}{\partial x}=\lambda\frac{\partial x^2+xy+y^2-\frac{13}{4}}{\partial y} \\ x^2+xy+y^2-\frac{13}{4}=0 \end{cases} \Rightarrow \begin{cases} y-1=\lambda(2x+y) \\ x-1=\lambda(x+2y) \\ x^2+xy+y^2-\frac{13}{4}=0 \end{cases} \Rightarrow y-x=\lambda(x-y) \tag{2}$$

$\Rightarrow x-y=0$ or $\lambda=-1$

(a) x=y

$$\begin{cases} x=y \\ x^2+xy+y^2=\frac{13}{4} \end{cases} \Rightarrow x^2+x^2+x^2=\frac{13}{4} \Rightarrow x=\pm\sqrt{\frac{13}{12}}, y=\pm\sqrt{\frac{13}{12}} \tag{3}$$

$$\Rightarrow \begin{cases} f(\sqrt{\frac{13}{12}},\sqrt{\frac{13}{12}})=\frac{25}{12}-\sqrt{\frac{13}{3}} \text{ critical point} \\ f(-\sqrt{\frac{13}{12}},-\sqrt{\frac{13}{12}})=\frac{25}{12}+\sqrt{\frac{13}{3}} \text{ maximumvalue and point} \end{cases} \tag{4}$$

(b) $\lambda=-1$

$$\begin{cases} 2x+2y=1 \\ x^2+xy+y^2=\frac{13}{4} \end{cases} \Rightarrow \begin{cases} x=\frac{-3}{2}, y=2 \\ x=2, y=\frac{-3}{2} \end{cases} \begin{cases} f(\frac{-3}{2},2)=\frac{-5}{2} \\ f(2,\frac{-3}{2})=\frac{-5}{2} \end{cases} \tag{5}$$

$\Rightarrow$ minimum value and point

– score: Find and right max and critical points get 5 points or right mini points get 5 points. Find x=y or λ = -1 get 3 points.