微乙小考一 (2018/3/15)

以下的函數 $f(x,y)$ 都是 $f(x,y)=y+(x^2-4x+4)(x+y^2)$
共四題，前後關聯，務必小心作答。

1. (5%) 求 $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$.

sol: $f_x(x,y)=\dfrac{\partial f}{\partial x}=\dfrac{\partial y}{\partial x}+\dfrac{\partial(x^2-4x+4)(x+y^2)}{\partial x}$
$=0+\dfrac{\partial(x^2-4x+4)}{\partial x}(x+y^2)+(x^2-4x+4)\dfrac{\partial(x+y^2)}{\partial x}$
$=(2x-4)(x+y^2)+(x^2-4x+4)$
$=2(x-2)(x+y^2)+(x-2)^2$
$=(x-2)(3x+2y^2-2)$ 或 $3x^2+2xy^2-8x-4y^2+4.$

$f_y(x,y)=\dfrac{\partial f}{\partial y}=\dfrac{\partial y}{\partial y}+\dfrac{\partial(x^2-4x+4)(x+y^2)}{\partial y}$
$=1+\dfrac{\partial(x^2-4x+4)}{\partial y}(x+y^2)+(x^2-4x+4)\dfrac{\partial(x+y^2)}{\partial y}$
$=1+0+(x^2-4x+4)2y$
$=1+2y(x-2)^2$ 或 $1+2x^2y-8xy+8y.$

2. (5%) 求曲面 $z=f(x,y)$ 在點 $x=2,\ y=1$ 之切面方程式。

sol: $z=f(x,y)$在點(a,b)之切面方程為$z=f(a,b)+f_x(a,b)(x-a)+f_y(a,b)(y-b)$.
由1.帶入得 $f_x(2,1)=0$ 且 $f_y(2,1)=1$, 因此
切面為 $z=f(2,1)+f_x(2,1)(x-2)+f_y(2,1)(y-1)\Rightarrow z=y.$

3. (5%) 用線性逼近求 $f(1.9,1.2)$ 之近似值。

sol: 線性逼近在三維即是用切平面逼近, 因為$(1.9,1.2)$是$(2,1)$附近的點, 可以用2.得到之切面方程$z=y$逼近, 以$(1.9,1.2)$代入得 $f(1.9,1.2)\approx 1.2.$
或 $f(x+\Delta x,y+\Delta y)-f(x,y)=\Delta z\approx f_x\Delta x+f_y\Delta y$為一個線性逼近.
由2.用$(2,1)$去近似得$\Delta x=1.9-2=-0.1$ 且 $\Delta y=1.2-1=0.2,$
$f(1.9,1.2)=f(2-0.1,1+0.2)\approx f(2,1)+f_x(2,1)(-0.1)+f_x(2,1)(0.2)=1.2.$

4. (5%) 設 $z(u,v)=f(x(u,v),y(u,v))$，其中

$$x(u,v)=\frac{u^2+v^3}{u^v+v^u+1}\quad\text{and}\quad y(u,v)=\frac{v+2}{u}.$$

求 $\dfrac{\partial z}{\partial u}$ 及 $\dfrac{\partial z}{\partial v}$ 在 $u=3,\ v=1$ 之值。提示: 用連鎖法則簡化計算。

sol: $z=f(x(u,v),y(u,v))$
根據連鎖法, $z_u(u,v)=z_x(x,y)x_u(u,v)+z_y(x,y)y_u(u,v),$

同理 $z_v(u,v) = z_x(x,y)x_v(u,v) + z_y(x,y)y_v(u,v)$,

且$u = 3,\ v = 1 \Rightarrow x(3,1) = 2,\ y(3,1) = 1.$

所求為 $z_u(3,1) = z_x(2,1)x_u(3,1) + z_y(2,1)y_u(3,1)$ 以及 $z_v(3,1) = z_x(2,1)x_v(3,1) + z_y(2,1)y_v(3,1)$,

由2.知道 $z_x(2,1) = 0,\ z_y(2,1) = 1$, 然後 $y_u = \dfrac{\partial \frac{v+2}{u}}{\partial u} = (v+2)(-\dfrac{1}{u^2})$,

$y_v = \dfrac{\partial \frac{v+2}{u}}{\partial v} = \dfrac{1}{u}$, 故$z_u(3,1) = 0 + 1(-\dfrac{1+2}{3^2}) = -\dfrac{1}{3}$, $z_v(3,1) = 0 + 1(\dfrac{1}{3}) = \dfrac{1}{3}$.