

6.1 二重積分

習題解答 6.1.2.

(1)

$$\begin{aligned}\iint_{\Omega} (1-x^2-y^2) dA &= \int_0^3 \left(\int_0^1 1-x^2-y^2 dy \right) dx \\ &= \int_0^3 \left(y-x^2y-\frac{y^3}{3} \Big|_0^1 \right) dx = \int_0^3 -x^2 + \frac{2}{3} dx \\ &= -\frac{x^3}{3} + \frac{2}{3}x \Big|_0^3 = -7\end{aligned}$$

(2)

$$\begin{aligned}\iint_{\Omega} x \cos y dA &= \int_0^1 \left(\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos y dy \right) dx \\ &= \int_0^1 \left(x \sin y \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \right) dx = \int_0^1 2x dx = x^2 \Big|_0^1 = 1\end{aligned}$$

(3)

$$\begin{aligned}\iint_{\Omega} \frac{1}{1+x+y} dA &= \int_0^1 \int_0^2 \frac{1}{1+x+y} dy dx \\ &= \int_0^1 \ln(1+x+y) \Big|_0^2 dx \\ &= \int_0^1 \ln(3+x-\ln(1+x)) dx \\ &= ((x+3)\ln(x+3)-x) \Big|_0^1 - ((x+1)\ln(x+1)-x) \Big|_0^1 \\ &= 4\ln 4 - 1 - 3\ln 3 - 2\ln 2 + 1 - 0 \\ &= 6\ln 2 - 3\ln 3 \quad (\text{或 } \ln \frac{64}{27})\end{aligned}$$

(4)

$$\begin{aligned}\int_1^2 \left(\int_1^3 \frac{xy}{(x^2+y^2)^2} dy \right) dx &= \int_1^2 \left(\frac{-x}{2(x^2+y^2)} \Big|_1^3 \right) dx \\ &= \frac{-1}{2} \int_1^2 \frac{x}{x^2+9} - \frac{x}{x^2+1} dx \\ &= \frac{-1}{2} \left(\frac{1}{2} \ln|x^2+9| - \frac{1}{2} \ln|x^2+1| \right) \Big|_1^2 \\ &= \frac{1}{2} \ln 5 - \frac{1}{4} \ln 13\end{aligned}$$

習題解答 6.1.3.

$$\begin{aligned} & \int_1^3 \left(\int_1^2 \frac{x}{(x^2 + y^2)^2} dx \right) dy \\ &= \int_1^3 \left(\frac{-1}{2(x^2 + y^2)} \Big|_1^2 \right) dy = \frac{1}{2} \int_1^3 \frac{1}{y^2 + 1} - \frac{1}{y^2 + 4} dy \\ &= \frac{1}{2} \tan^{-1} y \Big|_1^3 - \frac{1}{2} \cdot \frac{1}{4} \int_1^3 \frac{1}{1 + (\frac{y}{2})^2} dy \\ &= \frac{1}{2} \tan^{-1} y \Big|_1^3 - \frac{1}{4} \tan^{-1} \frac{y}{2} \Big|_1^3 = \frac{1}{2} \tan^{-1} 3 - \frac{1}{2} \tan^{-1} 1 - \frac{1}{4} \tan^{-1} \frac{3}{2} + \frac{1}{4} \tan^{-1} \frac{1}{2} \end{aligned}$$

習題解答 6.1.4.

$$\begin{aligned} \iint_{\Omega} f(x, y) dA &= \int_a^b \int_c^d h(x) \cdot k(y) dy dx \\ &= \int_a^b h(x) \cdot \left(\int_c^d k(y) dy \right) dx = \left(\int_c^d k(y) dy \right) \cdot \left(\int_a^b h(x) dx \right) \end{aligned}$$

習題解答 6.1.8.

$$\begin{aligned} & \int_a^b \int_c^d k + mx + ny dy dx \\ &= \int_a^b \left(ky + mxy + n \frac{y^2}{2} \right) \Big|_c^d dx \\ &= \int_a^b k(d - c) + m(d - c)x + \frac{n}{2}(d^2 - c^2) dx \\ &= k(d - c)(b - a) + m(d - c) \frac{b^2 - a^2}{2} + \frac{n}{2}(d^2 - c^2)(b - a) \\ &= (b - a)(d - c) \left(k + m \frac{a + b}{2} + n \frac{c + d}{2} \right) = A(\Omega) \cdot f\left(\frac{a + b}{2}, \frac{c + d}{2}\right) \end{aligned}$$