5.3 多變數函數之連鎖法則

習題解答 **5.3.3.**

(1) $\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u}$， $\frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v}$， $\frac{\partial z}{\partial p}=\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial p}$， $\frac{\partial z}{\partial q}=\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial q}$

(2) $\frac{dz}{dt}=\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial u}\cdot\frac{du}{dt}+\frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial v}\cdot\frac{dv}{dt}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial p}\cdot\frac{dp}{dt}+\frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial q}\cdot\frac{dq}{dt}$

習題解答 **5.3.4.**

(1)

$$\begin{aligned}\frac{dz}{dt} &= ye^{xy}\cdot(-\sin t)+xe^{xy}\cdot\cos t\\ &= -\sin^2 t\cdot e^{\cos t\sin t}+\cos^2 t\cdot e^{\cos t\sin t}=\cos(2t)e^{\frac{1}{2}\sin(2t)}\end{aligned}$$

(2)

$$\begin{aligned}\frac{dz}{dt} &= \frac{-y}{x^2}\cdot\frac{1}{1+\frac{y^2}{x^2}}\cdot e^t+\frac{1}{x}\frac{1}{1+\frac{y^2}{x^2}}\cdot 2t\\ &= \frac{1}{e^{2t}+t^4}(-t^2\cdot e^t+e^t\cdot 2t)=\frac{e^t}{e^{2t}+t^4}(2t-t^2)\end{aligned}$$

(4)

$$\begin{aligned}\frac{dz}{dt} &= 2x\cdot(1\cdot(-\sin t)+1\cdot\cos t)+2y\cdot(1\cdot(-\sin t)-1\cdot\cos t)\\ &= 2(\cos t+\sin t)(\cos t-\sin t)-2(\cos t-\sin t)(\cos t+\sin t)=0\end{aligned}$$

習題解答 **5.3.7.**

將 y 視為 x 的隱函數 $y(x)$ 得

$$f(x,y(x))=C,\quad y(x_0)=y_0$$

等號兩邊同時對 x 微分得

$$\begin{aligned}&\frac{\partial f}{\partial x}(x,y(x))+\frac{\partial f}{\partial y}(x,y(x))\cdot\frac{dy}{dx}(x)=0\\ \Rightarrow\ &\frac{\partial f}{\partial x}(x_0,y_0)+\frac{\partial f}{\partial y}(x_0,y_0)\cdot\frac{dy}{dx}(x_0)=0\\ \Rightarrow\ &\frac{dy}{dx}(x_0)=-\frac{\frac{\partial f}{\partial x}(x_0,y_0)}{\frac{\partial f}{\partial y}(x_0,y_0)}\end{aligned}$$

習題解答 **5.3.8.**

由上知切線方程式為

$$y-y_0=y'(x_0)(x-x_0)=-\frac{\frac{\partial f}{\partial x}(x_0,y_0)}{\frac{\partial f}{\partial y}(x_0,y_0)}(x-x_0)$$

可化簡為

$$\frac{\partial f}{\partial x}(x_0,y_0)\,(x-x_0)+\frac{\partial f}{\partial y}(x_0,y_0)\,(y-y_0)=0$$