

2.3 切線與線性逼近

習題解答 2.3.5.

$$(1) \sqrt[3]{0.997} = (1 + (-0.003))^{\frac{1}{3}} \approx 1 + \frac{-0.003}{3} = 0.999$$

(5) 由線性逼近知若 $f(x) = \tan^{-1}x$, 則

$$\begin{aligned}\tan^{-1}(1+h) &= f(1+h) \approx f(1) + f'(1)h \\ &= \tan^{-1}1 + \frac{1}{2} \cdot h = \frac{\pi}{4} + \frac{h}{2}\end{aligned}$$

所以

$$\tan^{-1}1.03 \approx \frac{\pi}{4} + \frac{0.03}{2} = \frac{\pi}{4} + 0.015 \quad (\approx 0.8)$$

(6) 由線性逼近知若 $f(x) = \cos^{-1}x$, $f'(x) = -\frac{1}{\sqrt{1-x^2}}$, 則

$$\begin{aligned}\cos^{-1}(0+h) &= f(0+h) \approx f(0) + f'(0)h \\ &= \cos^{-1}0 - \frac{1}{1} \cdot h = \frac{\pi}{2} - h\end{aligned}$$

所以

$$\cos^{-1}0.002 \approx \frac{\pi}{2} - 0.002 \quad (\approx 1.569)$$

習題解答 2.3.6.

由習題 2.3.1 知 $\sin(0+x) \approx x$, 且在 $x=0$ 附近

$$\cos x = \sqrt{1 - \sin^2 x} \approx \sqrt{1 - x^2} \approx 1 - \frac{x^2}{2}$$

$\cos x$ 在 $x=0$ 的線性逼近為

$$\cos(0+x) \approx \cos 0 - \sin 0 \cdot x = 1$$

前者提供更多資訊.

習題解答 2.3.7.

令 $V(r) = Cr^4$, 則 $\Delta V \approx V'(r)\Delta r = 4Cr^3\Delta r$

$$\frac{\Delta V}{V} \approx \frac{4Cr^3\Delta r}{Cr^4} = 4\frac{\Delta r}{r}$$

當管徑擴張或縮小 $\lambda\%$ 時, 血通量會擴張或縮小 $(4\lambda)\%$.

習題解答 2.3.11.

對 $\sin x$ 做對 $x=0$ 的一階逼近的誤差式:

$$\sin(0+x) = 0 + 1 \cdot x + \frac{-\sin \xi}{2}x^2 \quad \Rightarrow \quad \sin x - x = \frac{-\sin \xi}{2}x^2$$

但因為

$$-1 \leq -\sin \xi \leq 1 \quad \Rightarrow \quad -\frac{x^2}{2} \leq \sin x - x \leq \frac{x^2}{2}$$