

## 1.5 $e$ 與自然對數

### 習題解答 1.5.1.

$$(1) \ln e = \log_e e = 1$$

$$(2) e^{\ln x} = e^{\log_e x} = x$$

$$(3) \log_a x = \frac{\log_e x}{\log_e a} = \frac{\ln x}{\ln a}$$

### 習題解答 1.5.2.

$$(1) \lim_{x \rightarrow \frac{1}{\lambda}} \frac{\ln x + \ln \lambda}{\lambda x - 1} = \lim_{x \rightarrow \frac{1}{\lambda}} \frac{\ln \lambda x}{\lambda x - 1} \stackrel{y=\lambda x}{=} \lim_{y \rightarrow 1} \frac{\ln y}{y - 1} = 1$$

$$(2) \lim_{x \rightarrow 0} \frac{\ln \sec x}{\tan^2 x} = \lim_{x \rightarrow 0} \frac{1}{2} \cdot \frac{\ln(\sec x)^2}{\sec^2 x - 1} \stackrel{y=\sec^2 x}{=} \lim_{y \rightarrow 1} \frac{1}{2} \cdot \frac{\ln y}{y - 1} = \frac{1}{2}$$

### 習題解答 1.5.3.

$$(1) \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^{\lambda x} = \left(\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x\right)^{\lambda} = e^{\lambda}$$

$$(2) \lim_{x \rightarrow \infty} \left(1 + \frac{\lambda}{x}\right)^x = \lim_{x \rightarrow \infty} \left(\left(1 + \frac{1}{\frac{x}{\lambda}}\right)^{\frac{x}{\lambda}}\right)^{\lambda} \stackrel{y=\frac{x}{\lambda}}{=} \lim_{y \rightarrow \pm\infty} \left(\left(1 + \frac{1}{y}\right)^y\right)^{\lambda} = e^{\lambda}$$