微乙小考二 (2017/3/23)

1. (7分) 求 $f(x,y) = xe^y + \cos(xy)$

   (a) (3分) 在點 $(2,0)$ 的梯度;

   (b) (4分) 在點 $(2,0)$ 沿著方向 $\overrightarrow{u} = (3,-4)$ 的方向導數。

sol: (a) $\frac{\partial f}{\partial x}(x,y) = e^y - y\sin(xy) \Rightarrow \frac{\partial f}{\partial x}(2,0) = 1$

$\frac{\partial f}{\partial y}(x,y) = xe^y - x\sin(xy) \Rightarrow \frac{\partial f}{\partial y}(2,0) = 2$

$\therefore \nabla f(2,0) = (1,2)$

(b) $\frac{\partial f}{\partial \vec{u}}(2,0) = \nabla f(2,0) \cdot (\frac{3}{5}, \frac{-4}{5}) = (1,2) \cdot (\frac{3}{5}, \frac{-4}{5}) = -1$

2. (6分) 求過曲面 $x^3 + e^{x+xy} - \sin(yz) = 1$ 上點 $(0,1,\pi)$ 的切面方程式。

sol: Let $f(x,y,z) = x^3 + e^{x+xy} - \sin(yz) - 1$

$\frac{\partial f}{\partial x}(x,y,z) = 3x^2 + (1+y)e^{x+xy} \Rightarrow \frac{\partial f}{\partial x}(0,1,\pi) = 2$

$\frac{\partial f}{\partial y}(x,y,z) = xe^{x+xy} - z\cos(yz) \Rightarrow \frac{\partial f}{\partial y}(0,1,\pi) = \pi$

$\frac{\partial f}{\partial z}(x,y,z) = -y\cos(yz) \Rightarrow \frac{\partial f}{\partial z}(0,1,\pi) = 1$

Hence the tangent plane is $2x + \pi(y-1) + (z-\pi) = 0$

3. (7分) 求函數 $f(x,y) = 3y^2 - 2y^3 - 3x^2 + 6xy$ 的所有極值點候選點並討論其性質(包括鞍點)。

sol: $\frac{\partial f}{\partial x}(x,y) = -6x + 6y = 0$ and $\frac{\partial f}{\partial y}(x,y) = 6y - 6y^2 + 6x = 0$

the critical points are $(0,0)$ and $(2,2)$

$\frac{\partial^2 f}{\partial x^2} = -6,\ \frac{\partial^2 f}{\partial x \partial y} = 6 - 12y,\ \frac{\partial^2 f}{\partial y^2} = 6$

$D(0,0) = -72 < 0 \Rightarrow (0,0)$ is saddle point

$D(2,2) = 72 > 0 \Rightarrow (2,2)$ is relative maximum