1052 微乙小考一 (2017/3/9)

1. (6%) 若 $f(x,y,z) = z\tan^{-1}(\frac{y}{x}) + \ln(x^2+y^4)$，求下列偏導函數 $\frac{\partial f}{\partial x}(x,y,z)$, $\frac{\partial f}{\partial y}(x,y,z)$, $\frac{\partial f}{\partial z}(x,y,z)$。

   sol:

$$\begin{aligned}
\frac{\partial f}{\partial x} &= z\frac{1}{1+(y/x)^2}\cdot\left(-\frac{y}{x^2}\right) + \frac{1}{x^2+y^4}\cdot 2x = \frac{-yz}{x^2+y^2} + \frac{2x}{x^2+y^4} \\
\frac{\partial f}{\partial y} &= z\frac{1/x}{1+(y/x)^2} + \frac{4y^3}{x^2+y^4} = \frac{xz}{x^2+y^2} + \frac{4y^3}{x^2+y^4} \\
\frac{\partial f}{\partial z} &= \tan^{-1}\frac{y}{x}
\end{aligned}$$

2. (7%) 求函數 $z = x\cos y - ye^x$ 在 $(0,0,0)$ 的切平面方程式。

   sol: 檢查 $(0,0,0)$ 的確在該函數圖形上: $0 = 0\cos 0 - 0\cdot e^0$.

   計算法向量:

$$\begin{aligned}
\frac{\partial z}{\partial x}(0,0) &= \cos y - ye^x\,\Big|_{(0,0)} = 1 \\
\frac{\partial z}{\partial y}(0,0) &= -x\sin y - e^x\,\Big|_{(0,0)} = -1.
\end{aligned}$$

   得到法向量為 $(1,-1,-1)$. 利用點法式得到切平面方程式

$$x - y - z = 0.$$

3. (7%) 若 $z = xy + y^2$, $x = r\sin(rs)$, $y = r^2 + \ln s$, 其中 r, s 為獨立變數。求偏導函數 $\frac{\partial z}{\partial r}$ 和 $\frac{\partial z}{\partial s}$ 在$(r,s) = (\frac{\pi}{2},1)$ 之值。(即計算 $\frac{\partial z}{\partial r}(\frac{\pi}{2},1)$ 和 $\frac{\partial z}{\partial s}(\frac{\pi}{2},1)$ 之值。)

   sol: 先直接計算 $z = z(x(r,s),y(r,s))$ 和 (r,s) 的關係式:

$$z(x(r,s),y(r,s)) = r\sin(rs)(r^2+\ln s) + (r^2+\ln s)^2.$$

$$\begin{aligned}
\frac{\partial z}{\partial r} &= \sin(rs)(r^2+\ln s) + rs\cos(rs)(r^2+\ln s) + r\sin(rs)\cdot 2r + 2(r^2+\ln s)\cdot 2r \\
\frac{\partial z}{\partial r}(\frac{\pi}{2},1) &= \frac{\pi^2}{4}(3+2\pi) \\
\frac{\partial z}{\partial s} &= r^2\cos(rs)(r^2+\ln s) + r\sin(rs)\frac{1}{s} + 2(r^2+\ln s)\cdot\frac{1}{s} \\
\frac{\partial z}{\partial s}(\frac{\pi}{2},1) &= \frac{\pi}{2}(\pi+1).
\end{aligned}$$