微乙小考二 (2016/10/13)

1. (7分) 設 $\lambda \in \mathbb{R}$，求 $\lim_{n\to\infty}\left(1+\frac{\lambda}{n}\right)^{n+1}$.

sol:

$$\begin{aligned}\lim_{n\to\infty}(1+\frac{\lambda}{n})^{n+1} &= \lim_{n\to\infty}\big[(1+\frac{\lambda}{n})^{\frac{n}{\lambda}}\big]^{\lambda}(1+\frac{\lambda}{n}) \\ &= \big[\lim_{n\to\infty}(1+\frac{\lambda}{n})^{\frac{n}{\lambda}}\big]^{\lambda}\cdot\lim_{n\to\infty}(1+\frac{\lambda}{n}) \\ &= e^{\lambda}\cdot 1 = e^{\lambda}\end{aligned}$$

2. (7分) 求 $\lim_{x\to 0}\frac{\cos x-1}{x^2}$.

sol:

$$\begin{aligned}\lim_{x\to 0}\frac{\cos x-1}{x^2} &= \lim_{x\to 0}\frac{1-2\sin^2\frac{x}{2}-1}{x^2} \\ &= \lim_{x\to 0}\frac{-2\sin^2\frac{x}{2}}{(\frac{x}{2})^2}\cdot\frac{1}{4} \\ &= -\frac{1}{2}\end{aligned}$$

3. (6分) 求 $\lim_{x\to 1}\frac{\sqrt[3]{x^2}-1}{x-1}$.

sol:

$$\begin{aligned}\lim_{x\to 1}\frac{\sqrt[3]{x^2}-1}{x-1} &= \lim_{x\to 1}\frac{\sqrt[3]{x^2}-1}{x-1}\cdot\frac{\sqrt[3]{x^4}+\sqrt[3]{x^2}+1}{\sqrt[3]{x^4}+\sqrt[3]{x^2}+1} \\ &= \lim_{x\to 1}\frac{x^2-1}{(x-1)(\sqrt[3]{x^4}+\sqrt[3]{x^2}+1)} \\ &= \lim_{x\to 1}\frac{x+1}{\sqrt[3]{x^4}+\sqrt[3]{x^2}+1} \\ &= \frac{2}{3}\end{aligned}$$