1051 微乙小考一 (2016/9/29)

1. (6%) 令 $f(x) = x^2 + x + 1$。說明在 $x \geq -\frac{1}{2}$ 時，該函數有反函數，並具體求出此反函數及其定義域與值域的範圍。

   sol: $f(x) = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$, when $x \geq -\frac{1}{2}$, $f(x)$ is increasing, therefore, the inverse function exists. domain of $f(x)$: $\{x \in R : x \geq -\frac{1}{2}\}$ and range of $f(x)$: $\{y \in R : y \geq \frac{3}{4}\}$.

$$\begin{aligned} y &= (x + \frac{1}{2})^2 + \frac{3}{4} \\ \Rightarrow y - \frac{3}{4} &= (x + \frac{1}{2})^2 \\ \Rightarrow \pm\sqrt{y - \frac{3}{4}} &= x + \frac{1}{2}, \, since\, x + \frac{1}{2} \geq 0. \end{aligned}$$

   Therefore, $x = -\frac{1}{2} + \sqrt{y - \frac{3}{4}}$. i.e., $f^{-1}(x) = -\frac{1}{2} + \sqrt{x - \frac{3}{4}}$, with domain $\{x \in R : x \geq \frac{3}{4}\}$, and range $\{y \in \mathbb{R} : y \geq -\frac{1}{2}\}$

2. (7%) 求 $\sin^{-1}\cos(\frac{2}{3}\pi)$.

   sol: $\sin^{-1}\cos(\frac{2}{3}\pi) = \sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$

3. (7%) 計算 $\tan\sec^{-1} x$.

   sol: Let $\theta = \sec^{-1} x$, where $\theta \in [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$.

   (i) When $\theta \in [0, \frac{\pi}{2})$, $x \geq 1$,

$$\begin{aligned} \tan(\theta) &= \sqrt{\sec^2\theta - 1}, \text{ since } \tan\theta > 0 \\ &= \sqrt{x^2 - 1} \end{aligned}$$

   (ii) When $\theta \in (\frac{\pi}{2}, \pi]$, $x \leq -1$,

$$\begin{aligned} \tan(\theta) &= -\sqrt{\sec^2\theta - 1}, \text{ since } \tan\theta < 0 \\ &= -\sqrt{x^2 - 1} \end{aligned}$$

   Therefore,

$$\tan\sec^{-1} x = \begin{cases} \sqrt{x^2 - 1} & if\;\; x \geq 1 \\ -\sqrt{x^2 - 1} & if\;\; x \leq -1 \end{cases}$$