微乙小考四 (2015/5/14)

1. (6%) 求解下列微分方程:

$$y'(t) + ty(t) = t^3.$$

sol:

$$\begin{aligned} y'(t) + ty(t) &= t^3 \\ (e^{\frac{1}{2}t^2} \cdot y)' &= t^3 e^{\frac{1}{2}t^2} \\ e^{\frac{1}{2}t^2} y &= (t^2 - 2)e^{\frac{1}{2}t^2} + c \\ \therefore y(t) &= t^2 - 2 + ce^{-\frac{1}{2}t^2} \end{aligned}$$

2. (7%) 請解下列初始值問題:

$$y'(t) + \frac{3}{t}y(t) = 1 \text{ and } y(1) = 2.$$

sol:

$$\begin{aligned} y'(t) + \frac{3}{t}y(t) &= 1, \text{ and } y(1) = 2 \\ (t^3 y)' &= t^3 \\ t^3 y &= \frac{1}{4}t^4 + c \\ \because y(1) = 2 \Rightarrow c &= \frac{7}{4} \\ \therefore y(t) &= \frac{t^4 + 7}{4t^3} \end{aligned}$$

3. (7%) 求解下列微分方程:

$$y'(t) = y^2(t)(1 - 3y(t)).$$

sol:

$$\begin{aligned} \text{When } y' = 0 \Rightarrow y(t) &= 0 \text{ or } y(t) = \frac{1}{3} \\ y' &= y^2(1 - 3y) \\ \int \frac{1}{y^2(1-3y)} dy &= \int 1 dt \\ \int (\frac{3}{y} + \frac{1}{y^2} + \frac{9}{1-3y}) dy &= \int 1 dt \\ \ln\left|\frac{y}{1-3y}\right|^3 - \frac{1}{y} &= t + c \end{aligned}$$