1. (18%) Test the series for absolute convergence, conditional convergence or divergence.

(a) $\sum\limits_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$. (b) $\sum\limits_{n=1}^{\infty} (-1)^n \tan \frac{1}{n}$. (c) $\sum\limits_{n=1}^{\infty} \frac{(-1)^n}{\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{n^2}}$

Solution:

(a) (Total: **6 points**)

Step (1): Apply Integral Test to $\sum\limits_{n=2}^{\infty} \left| \frac{(-1)^n}{n(\ln n)^2} \right| = \sum\limits_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ (**3 points**).

Step (2): Correctly calculate the integral $\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx = \left. \frac{-1}{\ln x} \right|_2^{\infty} = \frac{1}{\ln 2}$ (**3 points**).

Step (3): Thus by Integral Test, $\sum\limits_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$ is *absolutely convergent.*

Grading Policies:

(1) As long as you applied Integral Test, you are granted **3 points** regardless of the correctness of your calculation of the integral $\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx$.

(2) If your calculation of the integral $\int_2^{\infty} \frac{1}{x(\ln x)^2}\, dx$ is wrong, **1 or 2 points** is granted depending on how many errors you make in that calculation.

(3) If you correctly proved that "$\sum\limits_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$ is convergent" by Alternating Series Test, you are also granted **3 points**. However, these 3 points do not stack with points granted from Step (1) or Step (2).

(b) (Total: **6 points**)

Step (1): Apply Limit Comparison Test to $\sum\limits_{n=1}^{\infty} \left| (-1)^n \tan \frac{1}{n} \right| = \sum\limits_{n=1}^{\infty} \tan \frac{1}{n}$ to compare it with $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ (**1 points**).

Step (2): Correctly derive the limit: $\lim\limits_{n\to\infty} \frac{\tan \frac{1}{n}}{\frac{1}{n}} = 1$ (**1 points**).

Step (3): Correctly state that $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ is divergent. (**1 points**).

Step (4): Thus by Limit Comparison Test, $\sum\limits_{n=1}^{\infty} \tan \frac{1}{n}$ is *divergent.*

Step (5): Apply Alternating Series Test to $\sum\limits_{n=1}^{\infty} (-1)^n \tan \frac{1}{n}$. (**1 points**).

Step (6): Correctly state that: $\lim\limits_{n\to\infty} \tan \frac{1}{n} = 0$ (**1 points**).

Step (7): Correctly state that: $\tan \frac{1}{n}$ is decreasing. (**1 points**).

Step (8): Thus by Alternating Series Test, $\sum\limits_{n=1}^{\infty} (-1)^n \tan \frac{1}{n}$ is *convergent.*

Step (9): Therefore, $\sum\limits_{n=1}^{\infty} (-1)^n \tan \frac{1}{n}$ is *conditionally convergent.*

Grading Policies:

Step (1) to Step (4) can be replaced by the following:

Step (1'): Apply Comparison Test to $\sum\limits_{n=1}^{\infty} \tan \frac{1}{n}$ to compare it with $\sum\limits_{n=1}^{\infty} \frac{1}{n}$ (**1 points**).

Step (2'): Correctly state that: $\tan \frac{1}{n} > \frac{1}{n},\ \forall n$ (**1 points**).

Step (3'): Correctly state that $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent. (**1 points**).

Step (4'): Thus by Comparison Test, $\sum_{n=1}^{\infty} \tan \frac{1}{n}$ is *divergent.*

(c) (Total: **6 points**)

Step (1): Correctly state that: $\lim_{n\to\infty} \frac{(-1)^n}{\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{n^2}} \neq 0$. (**6 points**). (For example, by p-series, $p = 2 > 1$, we have $\lim_{n\to\infty}\left(\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{n^2}\right) = \sum_{k=1}^{\infty}\frac{1}{k^2}$ is finite. So the above limit follows.)

Step (2): Thus by Test for Divergence, $\sum_{n=1}^{\infty} \frac{(-1)^n}{\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{n^2}}$ is *divergent.*

Grading Policies:

(1) If you only correctly proved the divergence of $\sum_{n=1}^{\infty} \frac{1}{\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{n^2}}$, you are granted **3 points**.

(2) If you applied Alternating Series Test and claimed that "because $\lim_{n\to\infty} \frac{1}{\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{n^2}} \neq 0$, i.e., the conditions for Alternating Series Test is not satisfied, therefore $\sum_{n=1}^{\infty} \frac{(-1)^n}{\frac{1}{1^2}+\frac{1}{2^2}+\cdots+\frac{1}{n^2}}$ is *divergent*", you will be *deducted* **3 points** because the logic is incorrect.

2. (10%)

(a) Find the Maclaurin series for $\cos^{-1} x$. (Write down the general term explicitly.)

(b) What is the radius of convergence of the series in (a).

(c) Let $f(x) = \cos^{-1}(x^2)$. Find $f^{(10)}(0)$.

Solution:

(a) Observe that $\frac{\mathrm{d}\cos^{-1}x}{\mathrm{d}x} = -\frac{1}{\sqrt{1-x^2}}$, so it suffices to find the Maclaurin series for $\frac{1}{\sqrt{1-x^2}}$.

$$\begin{aligned}\frac{1}{\sqrt{1-x^2}} &= \left(1-x^2\right)^{-1/2} = \sum_{n=0}^{\infty}\binom{\frac{-1}{2}}{n}\left(-x^2\right)^n \\ &= 1 + \frac{\left(\frac{-1}{2}\right)}{1!}\left(-x^2\right) + \frac{\left(\frac{-1}{2}\right)\cdot\left(\frac{-3}{2}\right)}{2!}\left(-x^2\right)^2 + \cdots + \\ &\quad \frac{\left(\frac{-1}{2}\right)\cdot\left(\frac{-3}{2}\right)\cdot\cdots\cdot\left(\frac{-1}{2}-n+1\right)}{n!}\left(-x^2\right)^n + \cdots \\ &= 1 + \frac{1}{2\cdot 1!}x^2 + \frac{1\cdot 3}{2^2\cdot 2!}x^4 + \cdots + \frac{1\cdot 3\cdot\cdots\cdot(2n-1)}{2^n\cdot n!}x^{2n} + \cdots \\ &= 1 + \sum_{n=1}^{\infty}\frac{1\cdot 3\cdot\cdots\cdot(2n-1)}{2^n\cdot n!}x^{2n} \\ &= \sum_{n=0}^{\infty}\frac{(2n)!}{\left(2^n\cdot n!\right)^2}x^{2n} \quad \text{(another expression)}\end{aligned}$$

Thus

$$\cos^{-1}x = \int -\sum_{n=0}^{\infty}\frac{(2n)!}{\left(2^n\cdot n!\right)^2}x^{2n}\mathrm{d}x = -\sum_{n=0}^{\infty}\frac{(2n)!}{\left(2^n\cdot n!\right)^2(2n+1)}x^{2n+1} + C$$

Now since $\cos^{-1} 0 = \frac{\pi}{2}$, we have $C = \frac{\pi}{2}$. Therefore

$$\cos^{-1}x = \frac{\pi}{2} - \sum_{n=0}^{\infty}\frac{(2n)!}{\left(2^n\cdot n!\right)^2(2n+1)}x^{2n+1}$$

(b) $R = 1$

(c) By (a), we have $\cos^{-1}(x^2) = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(2n)!}{(2^n \cdot n!)^2(2n+1)} (x^2)^{2n+1} = \frac{\pi}{2} - \sum_{n=0}^{\infty} \frac{(2n)!}{(2^n \cdot n!)^2(2n+1)} x^{4n+2}$. Thus $f^{(10)}(0)$ is obtained when $n = 2$, and then $f^{(10)}(0) = -\frac{3}{40} \cdot 10!$.

評分標準

(1) 三題配分分別是6分、2分和2分。

(2) 組合數沒展開扣1分;忘了常數項$\frac{\pi}{2}$扣1分。

(3) a小題微分微錯最多得4分。

(4) c小題須答案達一定程度才給分。

3. (8%) Find the interval of convergence of the series $\sum_{n=0}^{\infty} (n+3)x^n$, and compute the sum.

Solution:
We can use the ratio test to find the radius of convergence *(1 point)*, since

$$\lim_{n\to\infty} \frac{n+4}{n+3} = 1,$$

the radius is 1 *(1 point)*. To get the inteval of convergence , we need to check the end points, 1 and −1. Since

$$\lim_{n\to\infty} (n+3)$$

and

$$\lim_{n\to\infty} (-1)^n (n+3)$$

are both nonzero, the series is not convergent at −1 and 1. So the inteval of convergence is $[-1, 1]$ *(2 points)*.
To get the sum, we can find what $\Sigma_{n=0}^{\infty} 3x^n$ and $\Sigma_{n=0}^{\infty} nx^n$ are. Since $\Sigma_{n=0}^{\infty} x^n = \frac{1}{1-x}$ *(1 point)*,

$$\Sigma_{n=0}^{\infty} 3x^n = \frac{3}{1-x}.$$

And if we put $S = \Sigma_{n=0}^{\infty} nx^n$,

$$(1-x)S = x + x^2 + x^3 + \cdots = \frac{x}{1-x}$$

$$S = \frac{x}{(1-x)^2} \quad \textit{(2 points)}.$$

So the sum of the series is

$$\frac{3}{1-x} + \frac{x}{(1-x)^2} \quad \textit{(1 point)}.$$

4. (8%) Compute the sum of the series

$$S = (1)(\frac{1}{2}) + (1 - \frac{1}{3})(\frac{1}{2})^3 + (1 - \frac{1}{3} + \frac{1}{5})(\frac{1}{2})^5 + (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7})(\frac{1}{2})^7 + (1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9})(\frac{1}{2})^9 + \cdots.$$

(Hint: imitate the method of deriving the sum of a geometric series.)

Solution:
Let the sum be S, then

$$(1 - \frac{1}{4})S = \frac{3}{4}S = \frac{1}{2} - \frac{1}{3}(\frac{1}{2})^3 + \frac{1}{5}(\frac{1}{2})^5 - \frac{1}{7}(\frac{1}{2})^7 + \frac{1}{9}(\frac{1}{2})^9 - \cdots \quad \textit{(2 points)}.$$

Let

$$f(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \frac{1}{9}x^9 - \cdots$$

,

$$f'(x) = 1 - x^2 + x^4 - x^6 + x^8 - \cdots = \frac{1}{1+x^2} \text{ (2 points)}.$$

Then

$$f(x) = \int f'(x)\,\mathrm{d}x = \arctan x + C$$

where the constant C is clearly 0 *(2 points)*. So

$$S = \frac{4}{3}\arctan\frac{1}{2} \text{ (2 points)}.$$

5. (8%) A curve consists of two pieces of curves:

C_1 : $\mathbf{r}(t) = (t^2+3t)\mathbf{i} + (t^3-4t+1)\mathbf{j},\ t \le 0,$

C_2 : $y = p(x),\ x > 0,$ where $p(x)$ is a polynomial of degree 2.

Find the polynomial $p(x)$ so that this curve is continuous and has continuous slope and continuous curvature.

Solution:

Let $p(x) = ax^2 + bx + c$ for some parameters a, b, and c to be specified.

Consider $x = 0$ when $t = 0$, this curve is continuous.

$$\mathbf{r}(t=0) =< 0^2 + 3\times 0, 0^3 - 4\times 0 + 1 >=< 0, 1 >, \tag{1}$$

and

$$< x = 0, y = p(x=0) >=< 0, a0^2 + b0 + c >=< 0, c > . \tag{2}$$

Let (1) equals (2), we get $c = 1$.

Get 1 points with (1) or the conclusion $c = 1$.

Consider $x = 0$ when $t = 0$, this curve has continuous slope.

$$\frac{dy}{dx}\Big|_{t=0} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}\Big|_{t=0} = \frac{3t^2-4}{2t+3}\Big|_{t=0} = \frac{-4}{3} \tag{3}$$

and

$$\frac{dy}{dx}\Big|_{x=0} = 2ax^2 + b\Big|_{x=0} = b \tag{4}$$

Let (3) equals (4), we get $b = -\frac{4}{3}$.

Get 1 points with (3) or $r'(0) =< 3, -4 >$.

Get 1 points with the conclusion $b = -\frac{4}{3}$.

Consider $x = 0$ when $t = 0$, this curve has continuous curvature.

$$\kappa = \frac{|\mathbf{r}' \times \mathbf{r}''|}{|\mathbf{r}'|^3} = \frac{|< 2t+3, 3t^2-4, 0 > \times < 2, 6t, 0 >|}{|< 2t+3, 3t^2-4, 0 >|^3} \tag{5}$$

$$= \frac{|12t^2 + 18t - 6t^2 + 8|}{(4t^2 + 12t + 9 + 9t^4 - 24t^2 + 16)^{\frac{3}{2}}} \tag{6}$$

Then,

$$\kappa\Big|_{t=0} = \frac{8}{25^{\frac{3}{2}}} = \frac{8}{125} \tag{7}$$

and

Get 2 points with (7).

$$\kappa\Big|_{x=0} = \frac{|p''(x)|}{(1+p'(x)^2)^{\frac{3}{2}}}\Big|_{x=0} = \frac{|2a|}{(1+(2ax+b)^2)^{\frac{3}{2}}}\Big|_{x=0} = \frac{|2a|}{(1+b^2)^{\frac{3}{2}}} \tag{8}$$

Let (7) equals (8), we get $|a| = (\frac{8}{125})(\frac{125}{27})(\frac{1}{2}) = \frac{4}{27}$.

Get 2 points with (8).

Two polynomials of degree 2 are our solutions.

$$p_1(x) = \frac{4}{27}x^2 - \frac{4}{3}x + 1 \tag{9}$$

$$p_2(x) = -\frac{4}{27}x^2 - \frac{4}{3}x + 1 \tag{10}$$

Get 1 points with two solutions.

6. (12%) Find the limit, if it exists, or show that the limit does not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{x^5+x^2y^3}{x^4+y^6}$.   (b) $\lim_{(x,y,z)\to(0,0,0)} \frac{e^{xyz}-1}{x^2+y^2+z^2}$.

Solution:

(a) The limit does not exist, because the limit approaches different values along with $x = 0$ and $x^2 = y^3$.

Along with $x = 0$,

$$\lim_{y\to 0} \frac{0^5 + 0^2y^3}{0^4 + y^6} = \lim_{y\to 0} \frac{0}{y^6} = 0, \tag{11}$$

for all $y \neq 0$.

Get 3 points with one of limit value like equation (11).

Along with $x^2 = y^3$,

$$f(x, y) = f(y^{3/2}, y) = \frac{y^{15/2} + y^6}{y^6 + y^6} = \frac{y^{3/2} + 1}{2} \tag{12}$$

$$\lim_{y\to 0} \frac{y^{3/2} + 1}{2} = \frac{0 + 1}{2} = \frac{1}{2} \tag{13}$$

Get another 3 points with another limit value like equation (13).

(b) The limit approaches to zero.

Case 1: $xyz = 0$. Because $(x, y, z) \to (0, 0, 0)$ means $(x, y, z) \neq (0, 0, 0)$ and $x^2 + y^2 + z^2 \neq 0$. Then,

$$f(x, y, z) = \frac{e^0 - 1}{x^2 + y^2 + z^2} = 0. \tag{14}$$

Case 2: $xyz \neq 0$.

Transfer Cartesian coordinates to spherical coordinate.

$$x = r \sin\theta \cos\phi,$$
$$y = r \sin\theta \sin\phi,$$
$$z = r \cos\theta, \tag{15}$$

where $r > 0$ because of $xyz \neq 0$. Then,

$$f(x, y, z) = \frac{\exp\{r^3\delta\} - 1}{r^2}, \tag{16}$$

where $\delta = \sin^2\theta \cos\phi \sin\phi \cos\theta$. By L'Hospital's Rule,

$$\lim_{r\to 0} \frac{\exp\{r^3\delta\} - 1}{r^2} = \lim_{r\to 0} \frac{\exp\{r^3\delta\}3r^2\delta}{2r} == \lim_{r\to 0} r(\frac{3\delta \exp\{r^3\delta\}}{2}) = 0, \tag{17}$$

for all δ is finite.

Get 1 point if you conclude the limit 0 only through a specific direction like $x = 0$, $y = 0$, $x = z = 0$, or $x = y = z$, ...

Get 5 points if you proof the limit 0 by transforming spherical coordinate but with only wrong spherical expression.

Get 0 point if you try to proof the limit 0 with wrong logical derivations.

7. (10%) Let $f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2}, & \text{for } (x,y) \neq (0,0) \\ 0, & \text{for } (x,y) = (0,0). \end{cases}$

(a) Find $f_x(x,y)$ and $f_y(x,y)$.

(b) Are the functions f_x and f_y continuous at (0,0)?

Solution:

(a) (3pts) For $(x,y) \neq (0,0)$,

$$f_x(x,y) = \frac{3x^2}{x^2+y^2} - \frac{2x(x^3-y^3)}{(x^2+y^2)^2} = \frac{x^4 + 3x^2y^2 + 2xy^3}{(x^2+y^2)^2},$$

$$f_y(x,y) = \frac{-3y^2}{x^2+y^2} - \frac{2y(x^3-y^3)}{(x^2+y^2)^2} = \frac{-y^4 - 3x^2y^2 - 2yx^3}{(x^2+y^2)^2}.$$

(3pts) For $(x,y) = (0,0)$,

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h} = \lim_{h\to 0} \frac{h^3}{h^2} \cdot \frac{1}{h} = 1,$$

$$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k) - f(0,0)}{k} = \lim_{k\to 0} -\frac{k^3}{k^2} \cdot \frac{1}{k} = -1.$$

(b) (4pts) If f_x, f_y is continuous at $(0,0)$, then $\lim_{(x,y)\to(0,0)} f_x$ and $\lim_{(x,y)\to(0,0)} f_y$ exist and their values equal 1 and -1 respectively. But along $x = 0$,

$$\lim_{(x,y)\to(0,0)} f_x(x,y) = \lim_{a\to 0} f_x(0,a) = \lim_{a\to 0} \frac{0}{a^4} = 0,$$

which conflicts to $f_x(0,0)$ computed in (a). Similarily along $y = 0$,

$$\lim_{(x,y)\to(0,0)} f_y(x,y) = \lim_{b\to 0} f_y(b,0) = \lim_{b\to 0} \frac{0}{b^4} = 0,$$

which isn't equal to $-1 (= f_y(0,0) = -1)$.

8. (8%) Find the tangent plane of the surface

$$\frac{4}{\pi} \arctan \frac{z}{2} = x^2 + \int_{xy}^{z} xy\sqrt{1+t^3}\,dt$$

at the point $(1,2,2)$.

Solution:

Let $F(x,y,z) = x^2 + xy\int_{xy}^{z} \sqrt{1+t^3}\,dt - \frac{4}{\pi}\arctan\frac{z}{2}$. Note that $\nabla F(x_0,y_0,z_0)$ is normal to the point (x_0,y_0,z_0) on the surface $F(x,y,z) = 0$. Thus we compute ∇F at first,

$$F_x = 2x + y\int_{xy}^{z} \sqrt{1+t^3}\,dt + xy(\sqrt{1+(xy)^3} \cdot (-y)), \quad \text{(2pts)}$$

$$F_y = x\int_{xy}^{z} \sqrt{1+t^3}\,dt + xy(\sqrt{1+(xy)^3} \cdot (-x)), \quad \text{(2pts)}$$

$$F_z = xy\sqrt{1+z^3} - \frac{4}{\pi} \cdot \frac{1}{2} \cdot \frac{1}{1+(\frac{z}{2})^2}. \quad \text{(2pts)}$$

Hence, $\nabla F(1,2,2) = (2 + 0 - 4\sqrt{1+2^3}, -2\sqrt{1+2^3}, 2\sqrt{1+2^3} - \frac{2}{\pi}(\frac{1}{1+1})) = (-10, -6, 6 - \frac{1}{\pi})$ (1pt), and the tangent plane at $(1,2,2)$ is $-10(x-1) - 6(y-2) + (6 - \frac{1}{\pi})(z-2) = 0$. (1pt)

9. (8%) Find all points at which the direction of fastest change of the function $f(x,y,z) = x^2 + y^2 + z^2 - 2x - 4y - 6z$ is $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$.

Solution:
When $\nabla f = 2(x-1, y-2, z-3)$ is parallel to $(1,2,3)$[1 pt]

correct ans: $\begin{cases} \frac{x-1}{1} = \frac{y-2}{2} = \frac{z-3}{3} \\ \{(t,2t,3t) : t \in \mathbb{R}\} \\ \{(t,2t,3t) : t \in \mathbb{R}\backslash\{1\}\} \end{cases}$ [8 pts]

Note, "fastest" means increasing or decreasing most drastically.

Thus if you only consider the case:

$\begin{cases} \{(t,2t,3t) : t \in \mathbb{R}, t \geq 0\} \\ \{(t,2t,3t) : t \in \mathbb{R}\backslash\{1\}, t \geq 0\} \end{cases}$ you'll get [7 pts]

If you only write one or two points, such as $(1,2,3)$, you'll get [2 pts]

10. (10%) Let $g(x,y) = 4x^3 - 13y^3 + 6x^2y + 3xy^2 - 12x^2 - 12xy - 30y^2$. Find the critical points of $g(x,y)$, and classify them.

Solution:
$\nabla f = 0$ [1 pt]
critical points: $(0,0), (2,0), (\frac{2}{3}, \frac{-4}{3}), (\frac{8}{3}, \frac{-4}{3})$ [1 pt for each]
If you mentioned $D(x,y)$ [1 pt]

$\begin{cases} (0,0), D(0,0) > 0, g_{xx} < 0, \text{local maximum} \\ (2,0), D(2,0) < 0, \text{saddle point} \\ (\frac{2}{3}, \frac{-4}{3}), D(\frac{2}{3}, \frac{-4}{3}) < 0, \text{saddle point} \\ (\frac{8}{3}, \frac{-4}{3}), D(\frac{8}{3}, \frac{-4}{3}) > 0, g_{xx} > 0, \text{local minimum} \end{cases}$ [1 pt for each]

11. (10%) Find the points on the intersection of the plane $x + y + 2z = 2$ and the paraboloid $z = x^2 + y^2$ that are nearest to and farthest from the origin.

Solution:
Set

$$f(x,y,z) = x^2 + y^2 + z^2$$
$$g(x,y,z) = x + y + 2z - 2$$
$$h(x,y,z) = x^2 + y^2 - z.$$

Assume that $\nabla f + \lambda_1 \nabla g + \lambda_2 \nabla h = 0$. Then we obtain

$$2x + \lambda_1 + 2\lambda_2 x = 0$$
$$2y + \lambda_1 + 2\lambda_2 y = 0$$
$$2z + 2\lambda_1 - \lambda_2 = 0.$$

We also have

$$x + y + 2z = 2$$
$$x^2 + y^2 - z = 0.$$

Hence we can yield critical points are $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ and $(-1,-1,2)$. Since $f(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) = \frac{3}{4}$ and $f(-1,-1,2) = 6$, we can conclude that $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ is the point on the intersection nearest to the origin and $(-1,-1,2)$ is the point on the intersection farthest to the origin.

評分標準：函數假設寫好以及 Largrange's multiplier 的使用方式有做說明，這裡佔4%，方程式列出，解方程部份基本上不看過程(但必須要寫)，不過中間若有明顯錯誤便會扣分(依錯的程度來斟酌)，最後要說明哪個點為最大值點，哪個點為最小值點，此處必須解釋原因，只有寫下結論者扣4%，解釋不夠詳細會斟酌扣分，而筆墨分為2%，但空白者與不相干的過程皆不給分。