EX.5

(a) \[ \int_{0}^{10} f(x)dx \approx R_5 = [f(2) + f(4) + f(6) + f(8) + f(10)] \Delta x \]
\[ = [-1 + 0 + (-2) + 2 + 4](2) = 3(2) = 6 \]

(b) \[ \int_{0}^{10} f(x)dx \approx L_5 = [f(0) + f(2) + f(4) + f(6) + f(8)] \Delta x \]
\[ = [3 + (-1) + 0 + (-2) + 2](2) = 2(2) = 4 \]

(c) \[ \int_{0}^{10} f(x)dx \approx M_5 = [f(1) + f(3) + f(5) + f(7) + f(9)] \Delta x \]
\[ = [0 + (-1) + (-1) + 0 + 3](2) = 1(2) = 2 \]

EX.17

On \([2, 6]\), \(\lim_{n \to \infty} \sum_{i=1}^{n} x_i \ln(1 + x_i^2) \Delta x = \int_{2}^{6} x \ln(1 + x^2)dx\).

EX.34

(a) \[ \int_{0}^{2} g(x)dx = \frac{1}{2} \cdot 4 \cdot 2 = 4 \text{ [area of a triangle]} \]

(b) \[ \int_{2}^{6} g(x)dx = -\frac{1}{2} \pi (2)^2 = -2\pi \text{ [negative of the area of a semicircle]} \]

(c) \[ \int_{2}^{7} g(x)dx = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2} \text{ [area of a triangle]} \]
\[ \int_{0}^{7} g(x)dx = \int_{0}^{2} g(x)dx + \int_{2}^{6} g(x)dx + \int_{6}^{7} g(x)dx \]
\[ = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi \]

EX.38

First note that \[ \int_{-5}^{5} (x - \sqrt{25 - x^2})dx = \int_{-5}^{5} xdx - \int_{-5}^{5} \sqrt{25 - x^2}dx. \]
By symmetry, the value of the first integral is 0 since the area under the curve \(y = x\) and above the \(x\)-axis from 0 to 5 equals the area below the \(x\)-axis and above \(y = x\) from \(-5\) to 0. The second integral can be interpreted as one half the area of a circle with radius 5; that is, \(\frac{1}{2} \pi (5)^2 = \frac{25}{2} \pi\). Thus, the value of the original integral is \(0 - \frac{25}{2} \pi = -\frac{25}{2} \pi\).
EX.47

\[
\int_{-2}^{2} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx \\
= \int_{-2}^{-1} f(x)dx + \int_{2}^{5} f(x)dx + \int_{2}^{5} f(x)dx - \int_{-2}^{-1} f(x)dx \text{ [Property 5]} \\
= \int_{-1}^{5} f(x)dx \text{ [Property 5]}
\]

EX.51

\[
\int_{0}^{3} f(x)dx \text{ is clearly less than } -1 \text{ and has the smallest value. The slope of the tangent line of } f \text{ at } x = 1, f'(1), \text{ has a value between } -1 \text{ and } 0, \text{ so it has the next smallest value. The largest value is} \\
\int_{3}^{8} f(x)dx, \text{ followed by} \int_{4}^{8} f(x)dx, \text{ which has a value about 1 unit less than} \int_{3}^{8} f(x)dx. \text{ Still positive,} \\
\text{but with a smaller value than} \int_{4}^{8} f(x)dx, \text{ is} \int_{0}^{8} f(x)dx. \text{ Ordering these quantities from smallest to largest gives us} \\
\int_{0}^{3} f(x)dx < f'(1) < \int_{0}^{8} f(x)dx < \int_{4}^{8} f(x)dx < \int_{3}^{8} f(x)dx
\]
or \( B < E < A < D < C \).

EX.57

If \(-1 \leq x \leq 1, \text{ then } 0 \leq x^2 \leq 1 \text{ and } 1 \leq 1 + x^2 \leq 2, \text{ so } 1 \leq \sqrt{1 + x^2} \leq \sqrt{2} \text{ and } 1 \cdot [1 - (-1)] \leq \\
\int_{-1}^{1} \sqrt{1 + x^2}dx \leq \sqrt{2} \cdot [1 - (-1)] \text{ by Property 8; that is, } 2 \leq \int_{-1}^{1} \sqrt{1 + x^2}dx \leq 2\sqrt{2}.

EX.65

We have \( \int_{a}^{b} x^2 dx = \frac{b^3 - a^3}{3} \) from Exercise 28.

\[
\sqrt{x^4 + 1} \geq \sqrt{x^4} = x^2, \\
\text{so} \int_{1}^{3} \sqrt{x^4 + 1}dx \geq \int_{1}^{3} x^2dx \text{ [Property 7]} \\
= \frac{1}{3} (3^3 - 1^3) = \frac{26}{3} \text{ [Exercise 28]}
\]
EX.68

(a) Since $-|f(x)| \leq f(x) \leq |f(x)|$, it follows from Property 7 that $\int_a^b |f(x)| \, dx \leq \int_a^b f(x) \, dx \leq \int_a^b |f(x)| \, dx$. Note that the definite integral is a real number, and so the following property applies: $-a \leq b \leq a \Rightarrow |b| \leq a$ for all real numbers $b$ and nonnegative numbers $a$.

(b) $\left| \int_0^{2\pi} f(x) \sin 2x \, dx \right| \leq \int_0^{2\pi} |f(x)\sin 2x| \, dx \leq \int_0^{2\pi} |f(x)| \, dx$ (by part (a))

EX.71

$$\lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^4}{n^3} = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{i^4}{n^2} \cdot \frac{1}{n} = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{i}{n}\right)^4 \cdot \frac{1}{n}.$$ At this point, we need to recognize the limit as being of the form $\lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x$, where $\Delta x = (1-0)/n = \frac{1}{n}$, $x_i = 0 + i\Delta x = \frac{i}{n}$, and $f(x) = x^4$. Thus, the definite integral is $\int_0^1 x^4 \, dx$. 

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