Section 4.6 Graphing with Calculus and Calculators

EX.9

The graph of \( f(x) \):

\[
f'(x) = -\frac{1}{x^2} - \frac{16}{x^3} - \frac{3}{x^4}, \quad f''(x) = \frac{2}{x^3} + \frac{48}{x^4} + \frac{12}{x^5}.
\]

Note that \( f \) is undefined at \( x = 0 \).

Intervals of increase: \((-8 - \sqrt{61}, -8 + \sqrt{61})\)

Intervals of decrease: \((-\infty, -8 - \sqrt{61}), (-8 + \sqrt{61}, 0), (0, \infty)\)

Intervals of concave upward: \((-12 - \sqrt{138}, -12 + \sqrt{138}), (0, \infty)\)

Intervals of concave downward: \((-\infty, -12 - \sqrt{138}), (-12 + \sqrt{138}, 0)\)

EX.11

1. The graph of \( x^2 \ln x \):

2. \[
\lim_{x \to 0^+} x^2 \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x^2}} = \lim_{x \to 0^+} \frac{1/x}{-2/x^3} = \lim_{x \to 0^+} \frac{-x^2}{2} = 0,
\]

so \( f(x) \to 0 \) as \( x \to 0^+ \). (Note that \( \ln x \) is undefined when \( x \leq 0 \).)

3. \( f'(x) = x + 2x \ln x, \quad f''(x) = 3 + 2 \ln x. \)

Minimum value: \(-\frac{1}{2e}\)

Intervals of concave upward: \((e^{-3/2}, \infty)\)

Intervals of concave downward: \((0, e^{-3/2})\)
EX.13

1. The graph of $x^{1/x}$:

2. $f = e^{\frac{1}{2} \ln x} \cdot \frac{1}{x} \ln x$ approaches $-\infty$ and 0 as $x$ approaches $0^+$ and $\infty$. Thus, $f$ approaches 0 and 1 as $x$ approaches $0^+$ and $\infty$.

3. $f'(x) = x^{-2+\frac{1}{2}} (1 - \ln x)$ has a root $x = e$ and $f'(x) > 0$ for $x < e$, $f'(x) < 0$ for $x > e$, therefore the maximum value is $e^{\frac{1}{2}}$ and the minimum value does not exist.

4. $f''(x) = x^{-4+\frac{1}{2}} (1 - 3x + 2(-1 + x) \ln x + \ln^2 x)$.
EX.28

(1) Omit.

(2) $c = -10$: 

\[ c = -1: \]

\[ c = 0: \]

\[ c = 1: \]

\[ c = 10: \]

(3) The distance of $x$-axis between maximum and minimum becomes closer as $c \uparrow$ and $c < 0$ ; while there is no maximum and minimum as $c \geq 0$. The inflection point is $(0, 0)$.

(4) $c = 0$. 

EX.37

$c > 0,$

$f \to 0 \text{ as } x \to \infty, \ f \to -\infty \text{ as } x \to -\infty$

$c < 0,$

$f \to \infty \text{ as } x \to \infty, \ f \to 0 \text{ as } x \to -\infty$

$f' = (1 - cx)e^{-cx}$ has a root $x = \frac{1}{c}$ and thus reaches extreme value $\frac{1}{c}e^{-1}$
EX.39

\[ f(x) = cx^4 - 2x^2 + 1; \quad f'(x) = 4x(cx^2 - 1) \]
has roots \( x = 0, \pm \sqrt{\frac{1}{c}} \) for positive \( c \) and thus has the minimum \( 1 - \frac{1}{c} \) and the local maximum 1. The point \( \left( \sqrt{\frac{1}{c}}, 1 - \frac{1}{c} \right) \) lies on the parabola \( y = 1 - x^2 \). Also the point \( (0, 1) \) lies on this parabola.