Section 2.7 Derivatives and Rates of Change

EX.8

Find an equation of the tangent line to the curve at the given point.

$$y = \frac{2x+1}{x+2}, \qquad (1,1)$$

(sol)

$m = \lim_{x\to 1} \frac{\frac{2x+1}{x+2}-1}{x-1} = \lim_{x\to 1} \frac{x-1}{(x-1)(x+2)} = \frac{1}{3}$

tangent line : $y - 1 = \frac{1}{3}(x-1)$

EX.10

(a) Find the slope of the tangent to the curve $y = \frac{1}{\sqrt{x}}$ at the point where $x = a$.

(b) Find equations of the tangent lines at the points $(1,1)$ and $(4, \frac{1}{2})$.

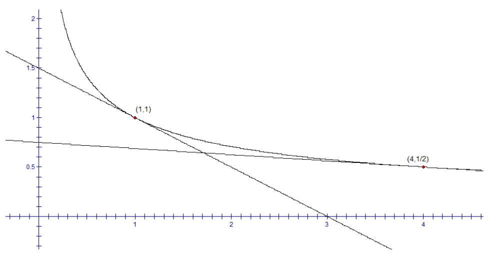
(c) Graph the curve and both tangents on a common screen.

(sol)

(a) $m = \lim_{x\to a} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}}}{x-a} = \lim_{x\to a} \frac{-\sqrt{x}+\sqrt{a}}{\sqrt{ax}(\sqrt{x}-\sqrt{a})(\sqrt{x}+\sqrt{a})} = \frac{-1}{2a\sqrt{a}}$

(b) $\begin{cases} a = 1 & \Rightarrow m = -\frac{1}{2} \\ a = 4 & \Rightarrow m = -\frac{1}{16} \end{cases} \Rightarrow \text{tangent line} : \begin{cases} y - 1 = -\frac{1}{2}(x-1) & \text{at } (1,1) \\ y - \frac{1}{2} = -\frac{1}{16}(x-4) & \text{at } (4, \frac{1}{2}) \end{cases}$

(c)



EX.20

If the tangent line to $y = f(x)$ at $(4,3)$ passes through the point $(0,2)$, find $f(4)$ and $f'(4)$.

(sol)

$f(4) = 3$

$m = \frac{3-2}{4-0} = \frac{1}{4} = f'(4)$

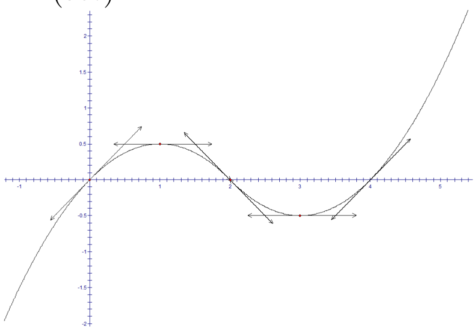
EX.22

Sketch the graph of a function g for which

$g(0) = g(2) = g(4) = 0, g'(1) = g'(3) = 0, g'(0) = g'(4) = 1$

$g'(2) = -1, \lim_{x\to\infty} g(x) = \infty$, and $\lim_{x\to-\infty} g(x) = -\infty$

(sol)



EX.31

Find $f'(a)$

$$f(x) = \sqrt{1-2x}$$

(sol)

$$\begin{aligned} f'(a) &= \lim_{x\to a} \frac{f(x)-f(a)}{x-a} \\ &= \lim_{x\to a} \frac{\sqrt{1-2x}-\sqrt{1-2a}}{x-a} = \lim_{x\to a} \frac{(1-2x)-(1-2a)}{(x-a)(\sqrt{1-2x}+\sqrt{1-2a})} \\ &= \lim_{x\to a} \frac{-2}{(\sqrt{1-2x}+\sqrt{1-2a})} = \frac{-1}{\sqrt{1-2a}} \end{aligned}$$

EX.35

Each limit represents the derivative of some function f at some number a. State such an f and a in each case.

$$\lim_{x\to 5} \frac{2^x - 32}{x-5}$$

(sol)

$f(x) = 2^x, a = 5$

EX.36

Each limit represents the derivative of some function f at some number a. State such an f and a in each case.

$$\lim_{x \to \frac{\pi}{4}} \frac{\tan x - 1}{x - \frac{\pi}{4}}$$

(sol)

$f(x) = \tan x, a = \frac{\pi}{4}$