Section 16.1 Vector Fields

EX.11

(IV). Because this is the only plot such that all the vectors in the first quadrant have positive \( x \) components and negative \( y \)-components. In addition the vectors become more parallel to the axes when we get closer to them.

EX.17

(III). Because this is the only plot such that all the vectors have fixed \( z \)-component while their \( x \)– and \( y \)-component change over space.

EX.18

(II). Because this is the only plot such that all the vectors point radially outward from the origin. In addition the vectors get shorter as we approach the origin.

EX.21

\[
\nabla f(x, y) = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} = (e^{xy} + xy e^{xy}) \mathbf{i} + x^2 e^{xy} \mathbf{j} = (1 + xy) e^{xy} \mathbf{i} + x^2 e^{xy} \mathbf{j}
\]

EX.24

\[
\nabla f(x, y, z) = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k} = \cos \left( \frac{y}{z} \right) \mathbf{i} - \frac{x}{z} \sin \left( \frac{y}{z} \right) \mathbf{j} + \frac{xy}{z^2} \sin \left( \frac{y}{z} \right) \mathbf{k}
\]

EX.26

\[
\nabla f(x, y) = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} = \frac{2x}{2\sqrt{x^2 + y^2}} \mathbf{i} + \frac{2y}{2\sqrt{x^2 + y^2}} \mathbf{j} = \frac{x}{\sqrt{x^2 + y^2}} \mathbf{i} + \frac{y}{\sqrt{x^2 + y^2}} \mathbf{j}
\]

The gradient vector field \( \nabla f \) is sketched in the following figure.
EX.29

(III). Because $\nabla f = 2x \mathbf{i} + 2y \mathbf{j}$, its gradient vectors should all point radially outward from the origin. Only the vector field in plot (III) satisfies this condition.

EX.30

(IV). Because $\nabla f = (2x + y) \mathbf{i} + x \mathbf{j}$, the $x$-components of its gradient vectors should vanish along the line $2x + y = 0$ while the $y$-components should be zero along the $y$-axis. Only the vector field in plot (IV) satisfies these conditions.

EX.31

(II). Because $\nabla f = 2(x+y) \mathbf{i} + 2(x+y) \mathbf{j}$, the gradient vector field should vanish along the line $x+y = 0$. Only the vector field in plot (II) satisfies this condition.

EX.32

(I). Because $\nabla f = \frac{x}{\sqrt{x^2 + y^2}} \cos(\sqrt{x^2 + y^2}) \mathbf{i} + \frac{y}{\sqrt{x^2 + y^2}} \cos(\sqrt{x^2 + y^2}) \mathbf{j}$, at each point its gradient vector should be parallel to the position vector. In addition the cosine factors imply that the directions of the gradient vectors should change periodically when we move away from the origin. Only the vector field in plot (I) satisfies these conditions.