Chapter 12 Problems Plus

EX.1

Since it is tightly packed. The straight line connects (0, 0, 0) to (1, 1, 1) must passes the center of two ball. See graph above

Then we have $3R + (\sqrt{2} - 1) R = \sqrt{2}$. We are done.

EX.7

(a) When object is static, by Newton’s first law, it is $N + F + W = 0$ which means $|N| = |W| \cos \theta_s$, $|W| = mg \sin \theta_s$ and hence $mg \sin \theta_s = \mu_s mg \cos \theta_s$. This implies $\mu_s = \tan \theta_s$.

(b) These two equations follow by Newton’s first law. One just expresses $H$ into two direction forces. Each force with other forces adding together would be zero.

(c) From above we have

\[ h_{\text{min}} \sin \theta + mg \cos \theta = n, \quad h_{\text{min}} \cos \theta + \mu_s n = mg \sin \theta \]

Substitute $n, \mu_s = \tan \theta_s$ into second equation we have

\[
mg \sin \theta = h_{\text{min}} \cos \theta + \mu_s (h_{\text{min}} \sin \theta + mg \cos \theta) \\
= h_{\text{min}} (\cos \theta + \mu_s \sin \theta) + \mu_s mg \cos \theta \\
= h_{\text{min}} (\cos \theta + \tan \theta_s \sin \theta) + \tan \theta_s mg \cos \theta.
\]

One has

\[ h_{\text{min}} = mg \frac{\sin \theta - \tan \theta_s \cos \theta}{\cos \theta + \tan \theta_s \sin \theta} = mg \tan (\theta - \theta_s). \tag{1} \]

When $\theta = \theta_s$, $h_{\text{min}} = 0$. That means we do not needs to add any additional forces to remain the block motionless. It makes sense to problem (b). As the angle goes to 90 degree. The equation above means that we need to add additional force $mg \tan (\frac{\pi}{2} - \theta_s)$ to let the block motionless. The equation (1) is reasonable for $\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$. 
