1. (6分) $\int \frac{\cos(\ln x)}{x} dx = ?$

   sol: Let $\ln x = y, dy = \frac{1}{x} dx$, then

$$\int \frac{\cos(\ln x)}{x} dx = \int \cos y\, dy = \sin y = \sin(\ln x).$$

2. (7分) $\int e^{2x} \sin x\, dx = ?$

   sol:

$$\int e^{2x} \sin x\, dx = e^{2x}(-\cos x) + 2\int e^{2x} \cos x\, dx$$

$$= -e^{2x} \cos x + 2(e^{2x} \sin x - 2\int e^{2x} \sin x\, dx)$$

$$\implies 5\int e^{2x} \sin x\, dx = -e^{2x} \cos x + 2e^{2x} \sin x$$

$$\implies \int e^{2x} \sin x\, dx = \frac{1}{5}(-e^{2x} \cos x + 2e^{2x} \sin x)$$

3. (7分)

   (a) $\frac{1}{(x-2)(x-1)^2} = \frac{a}{x-2} + \frac{b}{x-1} + \frac{c}{(x-1)^2}, a = ?, b = ?c = ?$ (3%)

   (b) $\int \frac{1}{(x-2)(x-1)^2} dx = ?$ (4%)

   sol: (a)

$$\frac{1}{(x-2)(x-1)^2} = \frac{a}{x-2} + \frac{b}{x-1} + \frac{c}{(x-1)^2}$$

$$\frac{a(x-1)^2 + b(x-1)(x-2) + c(x-2)}{(x-1)^2(x-2)}$$

$$\implies a(x-1)^2 + b(x-1)(x-2) + c(x-2) = 1 \quad (*)$$

   Let $x = 1$ in $(*) \implies c = -1$

   Let $x = 2$ in $(*) \implies a = 1$

   Consider the constant term in $(*)$, get $1 + 2b + 2 = 1 \implies b = -1$

   (b)

$$\int \frac{1}{(x-2)(x-1)^2} dx = \int (\frac{1}{x-2} + \frac{-1}{x-1} + \frac{-1}{(x-1)^2})$$

$$= \ln(|x-2|) - \ln(|x-1|) + \frac{1}{x-1}$$