

微乙小考三 (2014/10/30)

1. (6分) 令 $f(x) = (1 + x^2)^x$, 求 $f'(x)$.

sol:

$$\begin{aligned} f'(x) &= \frac{d}{dx}(1 + x^2)^x = \frac{d}{dx}e^{x \log(1+x^2)} \\ &= \frac{d}{dx}(x \log(1 + x^2))e^{x \log(1+x^2)} \\ &= \left(\log(1 + x^2) + \frac{2x^2}{1 + x^2} \right) e^{x \log(1+x^2)} \\ &= (1 + x^2)^x \log(1 + x^2) + 2x^2(1 + x^2)^{x-1}. \end{aligned}$$

2. (7分) 求曲線 $x^3 + y^3 = 3xy + 3$ 在 $(1, 2)$ 點之切線方程式.

sol: We only need compute the slope (i.e. dy/dx) at the point $(1, 2)$. To do so, we derivative the equation with respect to x and get

$$3x^2 + 3y^2 \frac{dy}{dx} = 3y + 3x \frac{dy}{dx}.$$

Now $(x, y) = (1, 2)$ gives

$$\left. \frac{dy}{dx} \right|_{(1,2)} = \frac{1}{3}.$$

Hence the tangent line is given by $y - 2 = \frac{1}{3}(x - 1)$.

3. (7分) 利用線性逼近法去估計 $\sqrt[4]{629}$ 之值.

sol: Let $f(x) = \sqrt[4]{x}$. We have $f(629) \sim f(625) + (629 - 625)f'(625)$ by using linear approximation. Hence

$$\sqrt[4]{629} \sim 5 + 4 \frac{1}{4\sqrt[4]{625^3}} = 5.008.$$