微乙小考二 (2014/10/16)

求下列極限:

1. (6分) $\lim\limits_{x\to 0}\frac{1-\cos(2x)}{x^2}$.

   sol: $\lim\limits_{x\to 0}\frac{1-\cos(2x)}{x^2}=\lim\limits_{x\to 0}\frac{2\sin^2 x}{x^2}=\lim\limits_{x\to 0}2(\frac{\sin x}{x})^2=2$

2. (7分) $\lim\limits_{n\to\infty}\left(\frac{n-4}{n-2}\right)^{3n}$.

   sol: $\lim\limits_{n\to\infty}(\frac{n-4}{n-2})^{3n}=\lim\limits_{n\to\infty}(1-\frac{2}{n-2})^{3n}=\lim\limits_{n\to\infty}[(1-\frac{2}{n-2})^{n-2}\cdot(1-\frac{2}{n-2})^2]^3=(e^{-2}\cdot 1^2)^3=e^{-6}$

3. (7分) $\lim\limits_{x\to 0}\frac{f\left(\frac{\sin x}{x}\right)^2}{g\left(\frac{\ln(1+x)}{x}\right)}$, **其中** f, g **為** $\mathbb{R}$ **上連續函數,且** $f(0)=3$, $f(1)=4$, $g(0)=5$, $g(1)=6$.

   sol: $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$ & $\lim\limits_{x\to 0}\frac{\ln(1+x)}{x}=\lim\limits_{x\to 0}\ln(1+x)^{\frac{1}{x}}=\ln e=1$

   Since f,g are continuous functions

   $\therefore\lim\limits_{x\to 0}\frac{f(\frac{\sin x}{x})^2}{g(\frac{\ln(1+x)}{x})}=\frac{[f(\lim\limits_{x\to 0}\frac{\sin x}{x})]^2}{g(\lim\limits_{x\to 0}\frac{\ln(1+x)}{x})}=\frac{[f(1)]^2}{g(1)}=\frac{8}{3}$