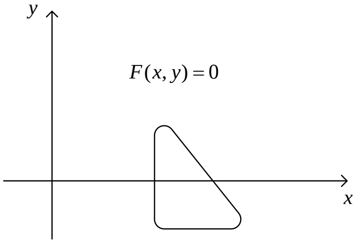
微乙小考一 (2014/10/2)

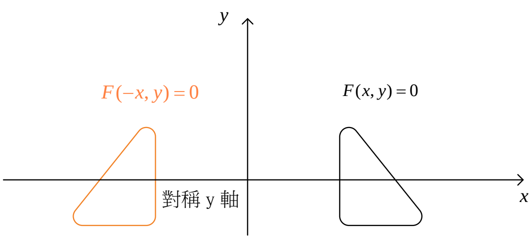
1. (7%) 說明 $y = 3^x - \frac{2}{3^x}$ 是 1-1 函數，並求其反函數。

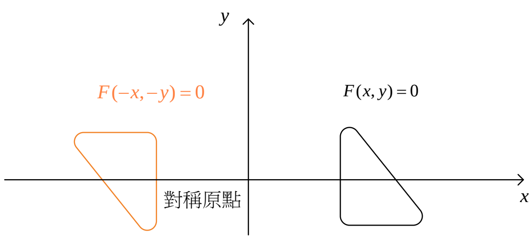
   sol: Let $x_1 > x_2$, then $3^{x_1} > 3^{x_2}$ & $\frac{2}{3^{x_1}} < \frac{2}{3^{x_2}}$.
   Hence, $y_1 = 3^{x_1} - \frac{2}{3^{x_1}} > 3^{x_2} - \frac{2}{3^{x_2}} = y_2$,
   that is, $y = 3^x - \frac{2}{3^x}$ is increasing$\Rightarrow$ which is one-to-one.
   To find its inverse function:
   $x = 3^y - \frac{2}{3^y}$, and let $A = 3^y$. Then we solve $x = A - \frac{2}{A}$.
   $x = A - \frac{2}{A} \Rightarrow A^2 - Ax - 2 = 0 \Rightarrow A = \frac{x+\sqrt{x^2+8}}{2} (\because A = 3^y > 0)$
   And hence,the inverse function is $y = \log_3 \frac{x+\sqrt{x^2+8}}{2}$.

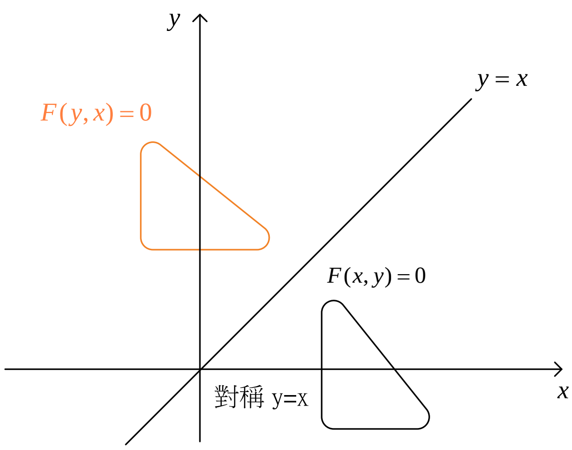
2. (7%) 已知 $F(x,y) = 0$ 之圖如下。畫出 $F(-x,y) = 0$，$F(-x,-y) = 0$ 及 $F(y,x) = 0$ 之圖。



   sol:







3. (6%) 求 $\sin^{-1}\left(\sin\frac{5}{6}\pi\right)$ 及 $\cos^{-1}\left(\cos\frac{5}{6}\pi\right)$。

sol: (1)$\sin^{-1}(\sin\frac{5\pi}{6}) = \sin^{-1}(\frac{1}{2}) = \frac{\pi}{6}(\because \frac{-\pi}{2} < \sin^{-1}x < \frac{\pi}{2})$

(2)$\cos^{-1}(\cos\frac{5\pi}{6}) = \cos^{-1}(-\frac{\sqrt{3}}{2}) = \frac{5\pi}{6}(\because 0 < \cos^{-1}x < \pi)$