# Solution of Exercise 2.3

## EX.22

$\lim_{u\to 2}\frac{\sqrt{4u+1}-3}{u-2} = \lim_{u\to 2}\frac{\sqrt{4u+1}-3}{u-2}\cdot\frac{\sqrt{4u+1}+3}{\sqrt{4u+1}+3} = \lim_{u\to 2}\frac{(4u+1)-9}{(u-2)(\sqrt{4u+1}+3)} = \lim_{u\to 2}\frac{4u-8}{(u-2)(\sqrt{4u+1}+3)} = \lim_{u\to 2}\frac{4}{\sqrt{4u+1}+3} = \frac{4}{3+3} = \frac{2}{3}$

## EX.26

$\lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t^2+t}\right) = \lim_{t\to 0}\left(\frac{1}{t}-\frac{1}{t(t+1)}\right) = \lim_{t\to 0}\frac{(t+1)-1}{t(t+1)} = \lim_{t\to 0}\frac{t}{t(t+1)} = \lim_{t\to 0}\frac{1}{t+1} = \frac{1}{0+1} = 1$

## EX.29

$\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}}-\frac{1}{t}\right) = \lim_{t\to 0}\frac{1-\sqrt{1+t}}{t\sqrt{1+t}} = \lim_{t\to 0}\frac{1-\sqrt{1+t}}{t\sqrt{1+t}}\cdot\frac{1+\sqrt{1+t}}{1+\sqrt{1+t}} = \lim_{t\to 0}\frac{1-(1+t)}{t\sqrt{1+t}(1+\sqrt{1+t})} = \lim_{t\to 0}\frac{-t}{t\sqrt{1+t}(1+\sqrt{1+t})} = \lim_{t\to 0}\frac{-1}{\sqrt{1+t}(1+\sqrt{1+t})} = \frac{-1}{\sqrt{1+0}(1+\sqrt{1+0})} = \frac{-1}{2}$

## EX.32

$\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h} = \lim_{h\to 0}\frac{\frac{x^2-(x+h)^2}{x^2(x+h)^2}}{h} = \lim_{h\to 0}\frac{-2hx-h^2}{hx^2(x+h)^2} = \lim_{h\to 0}\frac{-h(2x+h)}{hx^2(x+h)^2} = \lim_{h\to 0}\frac{-(2x+h)}{x^2(x+h)^2} = \frac{-(2x+0)}{x^2(x+0)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$

## EX.40

$-1 \le \sin(\pi/x) \le 1 \Rightarrow e^{-1} \le e^{\sin(\pi/x)} \le e^1 \Rightarrow \sqrt{x}/e \le \sqrt{x}e^{\sin(\pi/x)} \le \sqrt{x}e.$
Since $\lim_{x\to 0^+}(\sqrt{x}/e) = \lim_{x\to 0^+}\sqrt{x}e = 0$ ,we have $\lim_{x\to 0^+}[\sqrt{x}e^{\sin(\pi/x)}] = 0$ by the Squeeze Theorem.

# EX.45

Since $|x| = -x$ for $x < 0$,we have $\lim_{x\to 0^-}(\frac{1}{x} - \frac{1}{|x|}) = \lim_{x\to 0^-}(\frac{1}{x} - \frac{1}{-x}) = \lim_{x\to 0^-}\frac{2}{x}$ ,which does not exist since the denominator approaches 0 and the numerator does not.

# EX.46

Since $|x| = x$ for $x > 0$,we have $\lim_{x\to 0^+}(\frac{1}{x} - \frac{1}{|x|}) = \lim_{x\to 0^+}(\frac{1}{x} - \frac{1}{x}) = \lim_{x\to 0^+} 0 = 0$

# EX.52

## (a)

## (b)

### (i)

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = 0$, so $\lim_{x\to 0} f(x) = 0$

### (ii)

As $x \to (\frac{\pi}{2})^-$ , $f(x) \to 0$ , so $\lim_{x\to(\frac{\pi}{2})^-} f(x) = 0$

### (iii)

As $x \to (\frac{\pi}{2})^+$ , $f(x) \to -1$ , so $\lim_{x\to(\frac{\pi}{2})^+} f(x) = -1$

### (iv)

Since the answers in parts (ii) and (iii) are not equal , so $\lim_{x\to\frac{\pi}{2}} f(x)$ does not exist.

## (c)

$\lim_{x\to a} f(x)$ exists for all $a$ in the open interval $(-\pi, \pi)$ except $a = -\frac{\pi}{2}$ and $a = \frac{\pi}{2}$.

# EX.53

$\lim_{x\to 2^-} f(x) = 1-2 = -1$ , $\lim_{x\to 2^+} f(x) = 2-3 = -1$ , so $\lim_{x\to 2} f(x) = -1$. $f(2) = 2-2 = 0$ , hence $\lim_{x\to 2} f(x)$ exists but is not equal to $f(2)$.

# EX.59

Observe that $0 \le f(x) \le x^2$ for all $x$ , and $\lim_{x\to 0} 0 = 0 = \lim_{x\to 0} x^2$. So , by the Squeeze Theorem , we have $\lim_{x\to 0} f(x) = 0$.

# EX.63

Since the denominator approaches 0 as $x \to -2$, the limit will exist only if the numerator also approaches 0 as $x \to -2$. In order for this to happen, we need $\lim_{x\to -2}(3x^2 + ax + a + 3) = 0 \Leftrightarrow 3(-2)^2 + a(-2) + a + 3 = 0 \Leftrightarrow 12 - 2a + a + 3 = 0 \Leftrightarrow a = 15$. With $a = 15$, the limit becomes $\lim_{x\to -2} \frac{3x^2+15x+18}{x^2+x-2} = \lim_{x\to -2} \frac{3(x+2)(x+3)}{(x-1)(x+2)} = \lim_{x\to -2} \frac{3(x+3)}{x-1} = \frac{3(-2+3)}{-2-1} = \frac{3}{-3} = -1$.