17. Let $\epsilon > 0$ be given. 
\[ |\frac{1}{x+1} - \frac{1}{2}| = |\frac{1-x}{2(x+1)}| \]
If $|x - 1| < 1$, then $0 < x < 2$ and $1 < x + 1 < 3$, so that $|x + 1| > 1$. Let $\delta = \min(1, 2\epsilon)$. If $|x - 1| < \text{delta}$, then
\[ |\frac{1}{x+1} - \frac{1}{2}| = |\frac{x-1}{2(x+1)}| < \frac{2\epsilon}{2} = \epsilon \]
This establishes the required limit.

19. Let $\epsilon > 0$ be given. We have
\[ |\sqrt{x} - 1| = |\frac{x-1}{\sqrt{x}+1}| \leq |x - 1| < \epsilon \]
provided $|x - 1| < \delta = \epsilon$
This completes the proof.

21. We say that $\lim_{x \to a} f(x) = L$ if the following condition holds: for every number $\epsilon > 0$ there exists a number $\delta > 0$, depending on $\epsilon$, such that $x < -R$ implies $|f(x) - L| < \epsilon$.

22. We say that $\lim_{x \to -\infty} f(x) = L$ if the following condition holds: for every number $\epsilon > 0$ there exist a number $R > 0$, depending on $\epsilon$, such that $x < -R$ implies $|f(x)| < \epsilon$.

23. We say that $\lim_{x \to a} f(x) = -\infty$ if the following condition holds: for every number $B > 0$ there exist a number $\delta > 0$, depending on $B$, such that $0 < |x - a| < \delta$ implies $f(x) < -B$.

28. To be proved: that $\lim_{x \to 1^-} \frac{1}{x-1} = -\infty$. Proof: Let $B > 0$ be given. We have $\frac{1}{x-1} < -B$ if $0 > x - 1 > -\frac{1}{B}$, that is if $1-\delta < x < 1$, where $\delta = \frac{1}{B}$. This completes thr proof.

33. Let $\epsilon < 0$ be given. Since $\lim_{x \to a} f(x) = L$, there exists $\delta_1 > 0$ such that $|f(x) - L| < \frac{\epsilon}{2(1+|M|)}$ if $0 < |x - a| < \delta_1$. Since $\lim_{x \to a} g(x) = M$, there exists $\delta_2 > 0$ such that $|g(x) - M| < \frac{\epsilon}{2(1+|L|)}$ if $0 < |x - a| < \delta_2$. By Exercise 32, there exists $\delta_3 > 0$ such that $|g(x)| < 1+|M|$ if $0 < |x - a| < \delta_3$. Let $\delta = \min(\delta_1, \delta_2, \delta_3)$. If $|x - a| < \delta$, then
\[ |f(x)g(x) - LM| = |(f(x) - L)g(x) + Lg(x) - LM| \]
\[ \leq \left| (f(x) - L)g(x) \right| + \left| L(g(x) - M) \right| = |f(x) - L| |g(x)| + |L| |g(x) - M| \]
\[ < \frac{\varepsilon}{2(1+|M|)}(1+|M|) + \frac{|L|}{2(1+|M|)} \leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon \]

34. To be proved: if \( \lim_{x \to a} g(x) = M \) where \( M \neq 0 \), then there exists \( \delta > 0 \) such that if \( 0 < |x - a| < \delta \), then \( |g(x) - M| < \frac{|M|}{2} \) (since \( \frac{|M|}{2} \) is a positive number). This latter inequality implies that

\[ |M| = |g(x) + (M - g(x))| \leq |g(x)| + |g(x) - M| < |g(x)| + \frac{|M|}{2} \]

It follows that \( |g(x)| > |M| - \frac{|M|}{2} = \frac{|M|}{2} \), as required.

35. Let \( \epsilon > 0 \) be given. Since \( \lim_{x \to a} g(x) = M \neq 0 \), then exists \( \delta_1 > 0 \) such that \( |g(x) - M| < \frac{\epsilon |M|}{2} \) if \( 0 < |x - a| < \delta_1 \). By Exercise 34, there exists \( \delta_2 > 0 \) such that \( |g(x)| > \frac{|M|}{2} \) if \( 0 < |x - a| < \delta_3 \). Let \( \delta = \min(\delta_1, \delta_2) \). If \( 0 < |x - a| < \delta \), then

\[ \frac{1}{g(x)} - \frac{1}{M} = \frac{|M - g(x)|}{|M||g(x)|} < \frac{\epsilon}{2} \frac{|M|^2}{|M|^2} = \frac{\epsilon}{2} \]

This completes the proof.

37. Let \( \epsilon > 0 \) be given. Since \( f \) is continuous at \( L \), there exists a number \( \gamma > 0 \) such that if \( |y - L| < \gamma \), then \( |f(y) - f(L)| < \epsilon \). Since \( \lim_{x \to a} g(x) = L \), there exists \( \delta > 0 \) such that if \( 0 < |x - c| < \delta \), then \( |g(x) - L| < \gamma \). Taking \( y = g(x) \), if follows that if \( 0 < |x - c| < \delta \), then \( |f(g(x)) - f(L)| < \epsilon \), so that \( \lim_{x \to c} f(g(x)) = L \).