

Real Analysis Homework #6

Due 11/3

Do 1 & 2 from Homework # 5.

3. Define f by

$$f(x) = \begin{cases} 0, & x \in \mathbb{Q} \cap (0, 1), \\ [1/x]^{-1}, & x \in \mathbb{Q}^c \cap (0, 1), \end{cases}$$

where $[x]$ = integer part of x . Find $\int_0^1 f dx$.

4. Let \mathcal{C} be the Cantor set of $[0, 1]$. Define

$$f(x) = \begin{cases} 0, & x \in \mathcal{C}, \\ n, & x \text{ in the complementary interval of length } 3^{-n}. \end{cases}$$

Show that f is measurable and evaluate $\int_0^1 f(x) dx$.

5. Let (X, \mathcal{T}) be a second-countable topological space and (Y, d) any metric space. Show that the Borel σ -algebra in the product $X \times Y$ is the product σ -algebra of the Borel σ -algebras in X and in Y . (See Dudley p.123 for some hints.)