MACHINE LEARNING-BASED FORECASTING THE TREND OF COVID-19 OUTBREAK IN TAIWAN

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Abstract. The world has been suffering from the impact of COVID-19 pandemic since the virus was first detected in the late December of 2019. The economic and social disruption caused by the pandemic is devastating. Until now, even though several vaccines have been approved by WHO and a lot of countries have accelerated vaccine rollout in an effort to prevent further outbreak of the pandemic, the world is still under the threat of new variants of COVID-19 virus.

Unlike most countries, for the past one and a half years, Taiwan was virtually virus-free and people were doing business as usual. However, new cases of infection emerged dramatically in mid-May this year. To curb the spread of the virus, Taiwanese government imposed Level 3 epidemic alert on May 20, 2021. Level 3 epidemic alert is still in effect at the time of writing this paper. Out of curiosity, we are interested in predicting the current trend of Taiwan's pandemic combining a simple mathematical model and deep learning. Our model is a modification of multi-chain Fudan-CCDC models introduced in [PSY+20a, PSY+20b].

In [PSY+20a, PSY+20b], the spread of virus depends entirely on two parameters – infection rate and isolation rate. These two parameters are assumed to be piecewise constants in [PSY+20a, PSY+20b]. However, we were not able to make a satisfactory prediction of the COVID-19 pandemic outbreak in Taiwan using their models. Therefore, we consider more general parameters, not necessarily piecewise constants, in our model.

There are two major difficulties in predicting the epidemic with a mathematical model. One difficulty is that the only available data is the number of daily infected cases provided by the government. All other parameters and variables in the model are unknown. We are able to approximate the parameters and the unknown variables based on the mathematical model using the available data of daily infected cases. Another difficulty is that all parameters are unknown when we perform the prediction. Hence, it becomes impractical to use the mathematical model to do the forecasting. To overcome this difficulty, we make use of long short-term memory neural network (LSTM), a machine learning algorithm.

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1. Introduction

The COVID-19 pandemic has been the news headline for almost one year and a half since the virus was first reported in December, 2019. The new coronavirus is contagious and

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quickly spread globally. The pandemic has caused not only a significant loss of lives but also the stagnation of global economic. Never before have people experienced social lockdown for such a long period. Until now, even though several vaccines have been approved by WHO and a lot of countries have accelerated vaccine rollout in an effort to prevent further outbreak of the pandemic, the world is still under the threat of new variants of COVID-19 virus. The path to normality may be further delayed.

Unlike most countries, Taiwan was virtually virus-free for the past one and a half year thanks to prompt responses at the onset of global pandemic. However, the number of new infected cases suddenly arose in mid-May, 2021 (largely due to the α variant virus). It caught the government off guard. To prevent further spread of the virus, Taiwanese government imposed a Level 3 epidemic alert on May 20, 2021. Key restrictions in the alert include that all people must weak masks at all times outdoors, and indoor gatherings are limited to five people, while outdoor gatherings are restricted to ten. A wide range of business and public venues are to be closed, with the exception of essential services such as police departments, hospitals, and government buildings. All school classes are conducted online.

In order to better understand the outbreak of COVID-19 pandemic in Taiwan and to make a reasonable prediction of the trend, we propose a simple epidemic mathematical model. We are especially interested in the prediction when the Level 3 alert is likely to be lifted. Since last year, there are a large pool of mathematical literature discussing the COVID-19 pandemic from different aspects. It is not possible review all of them here. We only mention some of them which are closely related to our interests, [CCJL20a, CCJL20b, LJYC20, LSCC20, SCC20, SCCC20, SZCC20, SZY+20, XCY+21, YCL+20]. Our model is mainly inspired by the multi-chain Fudan-CCDC model [PSY+20a, PSY+20b], where the infection and the isolation rates play the important role in predicting the trend of the pandemic. We remark that a multi-chain Fudan-CCDC model (with constant infection and isolation rates) is indeed equivalent to a single-chain Fudan-CCDC model with piecewise constant parameters. However, we were not able to make a satisfactory prediction of the recent COVID-19 pandemic outbreak in Taiwan using their models with piecewise constant infection and isolation rates. Therefore, we want to propose a modified mathematical model and consider more general parameters, not necessarily piecewise constants, in the model.

The chance of successfully forecasting the spread of the virus depends essentially on how good the estimates of the parameters using the public available data. Nonetheless, the only available data is the number of daily infected cases released by the government. All other parameters and variables in the model are unknown. We can overcome this obstacle by fitting the parameters and the unknown variables based on the mathematical model using the available data of daily infected cases. However, in the prediction step, all parameters are still unknown. Hence, it becomes impractical to use the mathematical model to do the forecasting. To overcome this difficulty, we make use of long short-term memory neural network (LSTM), a machine learning algorithm.

In our approach, we divide the data obtained from the public data of the COVID-19 pandemic in two groups:

(i) Training set: these data are given to the network to train the system, i.e., adjusting the parameters in the model to minimize the fitting error;
(ii) Validation set: these data are used to measure the accuracy of the system and to halt the training when the error is minimized.

After training our system, we then perform the prediction based on the trained model with the parameters determined by the network.

It is worth to mention that one can perform of the forecasting of the pandemic by feeding only the number of daily infected cases into the LSTM algorithm without referring to the mathematical model. This is a type of unsupervised learning. In our approach, we will make use of the mathematical model to produce the training dataset and train the system based on this dataset. In other words, our method can be seen as a supervised learning. In later section, we will provide an evidence to convince the reader that our method outperforms the unsupervised learning.

2. Mathematical model

We are now ready to describe the mathematical model. Let p be the transition probability from infection-to-illness onset. According to the early transmission dynamics in Wuhan of the COVID-19 disease, which studied in [LGW+20] by Chinese Center for Disease Control and Prevention (CCDC), p can be approximated by a log-normal distribution $\mathrm{Lognormal}(\mu, \sigma^2)$ with $\mu = 1.417$ and $\sigma^2 = 0.4525$, that is,

$$p(t) = \frac{0.5977}{t} e^{-1.105(\ln t - 1.417)^2}. \tag{2.1}$$

Here we remark that

$$\mathbb{E}[p] \approx 5.172 \quad \text{and} \quad \int_{t \leq 14} p(t)\, dt \approx 0.965. \tag{2.2}$$

Therefore, in our model, it is reasonable to approximate (2.1) by

$$p(t) \approx \mathbb{1}_{t \leq 14} \frac{0.5977}{t} e^{-1.105(\ln t - 1.417)^2}.$$

Remark 2.1. According to the introduction of COVID-19 by Taiwan Centers for Disease Control (Taiwan CDC), the incubation period from infection with the COVID-19 to the onset of disease is 1 to 14 days (mostly 5 to 6 days), which agrees to (2.2).

The quantities used in the dynamics of the COVID-19 infection in our model are listed below:

$$\begin{cases} I_t: & \text{the cumulative infected people at time } t, \\ S_t: & \text{the number of symptomatic cases at time } t, \\ C_t: & \text{the cumulative confirmed cases at time } t, \\ Q_t: & \begin{array}{l}\text{the number of infected people who are in quarantine} \\ \text{but yet to be confirmed at time } t,\end{array} \\ H_t: & \begin{array}{l}\text{the number of infected people who are neither in} \\ \text{quarantine nor hospitalized at time } t,\end{array} \end{cases}$$

see Figure 2.1 for the relations between different quantities. We then denote

$$\begin{aligned} \Delta I_t &:= I_t - I_{t-1}, \\ \Delta C_t &:= C_t - C_{t-1}, \\ \Delta Q_t &:= Q_t - Q_{t-1}. \end{aligned}$$

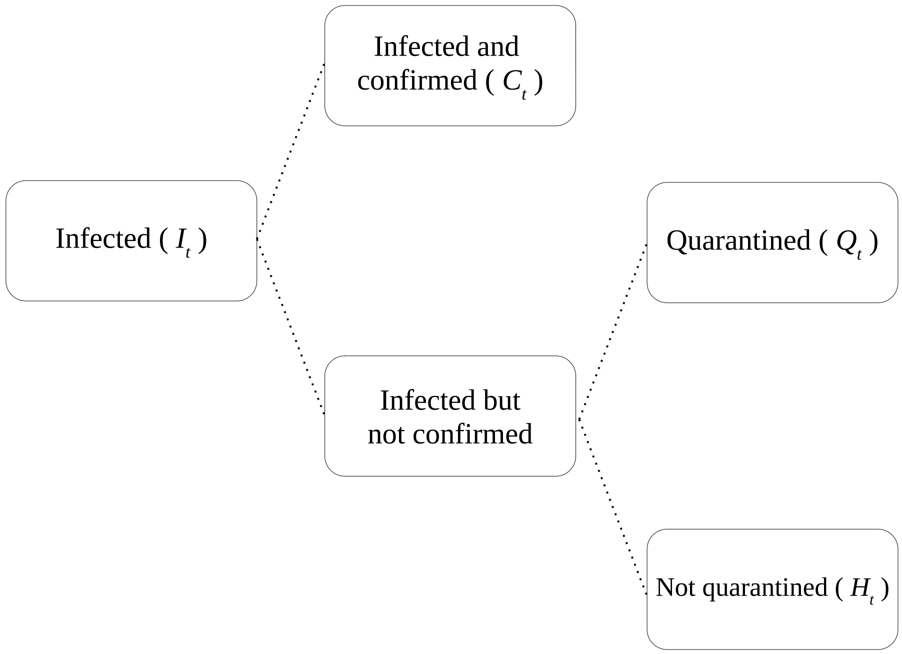

FIGURE 2.1. Relations of the quantities

Based on the observation in [LGW+20], the following relation is proposed:

$$S_t = \sum_{i=1}^{14} p_i \Delta I_{t-i}, \quad \text{where} \quad p_i = p(i). \tag{2.3}$$

Our model contains several key parameters:

$$\begin{cases} \beta_t : & \text{the infection rate,} \\ \gamma_t : & \text{the confirmation rate for symptomatic cases,} \\ \delta_t : & \text{the confirmation rate for asymptomatic cases,} \\ \ell_t : & \text{the isolation rate.} \end{cases}$$

We expect that $\delta_t \ll \gamma_t$ since, for asymptomatic cases, their Ct values (cycle threshold values) in RT-PCR tests are still in the range of being categorized as negative cases (see Figure 3.5.)

We assume that the infection chain begins at time $t = 16$. In this paper, we propose the following nonlocal linear COVID-19 epidemic model:

$$\begin{cases} \Delta I_t = \beta_t H_{t-1} & \text{, for all integers } t \geq 16, \\ \Delta C_t = \gamma_t S_t + \delta_t Q_{t-1} & \text{, for all integers } t \geq 16, \\ \Delta Q_t = \ell_t \Delta I_t - \Delta C_t & \text{, for all integers } t \geq 16, \\ H_t = I_t - C_t - Q_t & \text{, for all integers } t \geq 16. \end{cases} \tag{2.4}$$

We would like to give further explanations of the model (2.4). The first equation indicates that the increase of newly infected cases at t is equal to the infection rate β_t times the hidden cases H_{t-1} on the previous day $t-1$. The parameter β_t and variables I_t, H_t are unknown. The infection rate β_t depends on several factors such as face masks requirement, social distancing, and personal hygiene, etc. The second equation explains the increase of confirmed cases at t. It comes from two parts. The first part is the number of symptomatic cases that have positive tests (RT-PCR test). This part is determined by the confirmation rate γ_t and the number of symptomatic cases S_t. The second part is the number of confirmed positive cases that come from the asymptomatic people who are in quarantine at day $t-1$.

The third equation of (2.4) indicates the change of people in quarantine at time t. This quantity is equal to the isolation rate ℓ_t times the increase of infected cases at time t subtracts the confirmed cases at time t. The isolation rate ℓ_t depends on the government's policy in response to the severity of the pandemic, for example, the level of epidemic alert. Finally, the last equation defines the number of hidden cases.

3. Interpolation and model fitting

Sine all parameters and initial conditions of variables in (2.4) are not known, it is impossible to solve the equation in practice. The only data we know is the cumulative confirmed cases (or the daily confirmed cases). We arrange the time series data $\hat{\boldsymbol{C}} = \{\hat{C}_t\}_{t=16}^{15+d}$ of confirmed cases, where $d > 1$ is arbitrary depending on different scenario. Here we choose $d = 85$. To fit the model, we use the data (number of daily confirmed cases) from April 22, 2021 to July 15, 2021, which correspond to the indices $t = 16$ and $t = 15 + 85 = 100$, respectively. We remark that the choice of the starting index $t = 16$ is due to (2.3). In other words, the first significant data is the number of confirmed cases on April 22, 2021, i.e., $\hat{C}_{16}$. Therefore, it is reasonable to assume that

$$C_t = Q_t = 0, \quad \text{for all integers} \quad 1 \le t \le 15,$$

before the outbreak of the pandemic. Note that the time series of $\boldsymbol{I}, \boldsymbol{Q}, \boldsymbol{H}, \boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{\ell}$ are all unknown. Our strategy is to interpolate $\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{\ell}$ by minimizing the following objective quantity:

$$\|\boldsymbol{C} - \hat{\boldsymbol{C}}\|_{\ell^2}^2 = \sum_{t=16}^{15+d} |C_t - \hat{C}_t|^2, \quad d = 85. \tag{3.1}$$

We will minimize (3.1) by implementing the MATLAB® function `fminsearch`, which locates the minimum of a *unconstrained* multi-variable function using derivative-free method. Since the parameters $\boldsymbol{\beta}, \boldsymbol{\gamma}, \boldsymbol{\delta}, \boldsymbol{\ell}$ are non-negative, we have to use the function wisely[3]. In the work, we use the flow chart as described in Figure 3.1. The interpolation results are shown in Figure 3.2, Figure 3.3, Figure 3.4, and Figure 3.5.

[3] `https://puzhaokow1993.github.io/homepage/Publications/upload/COVID19_Taiwan_Program/main_program.html`

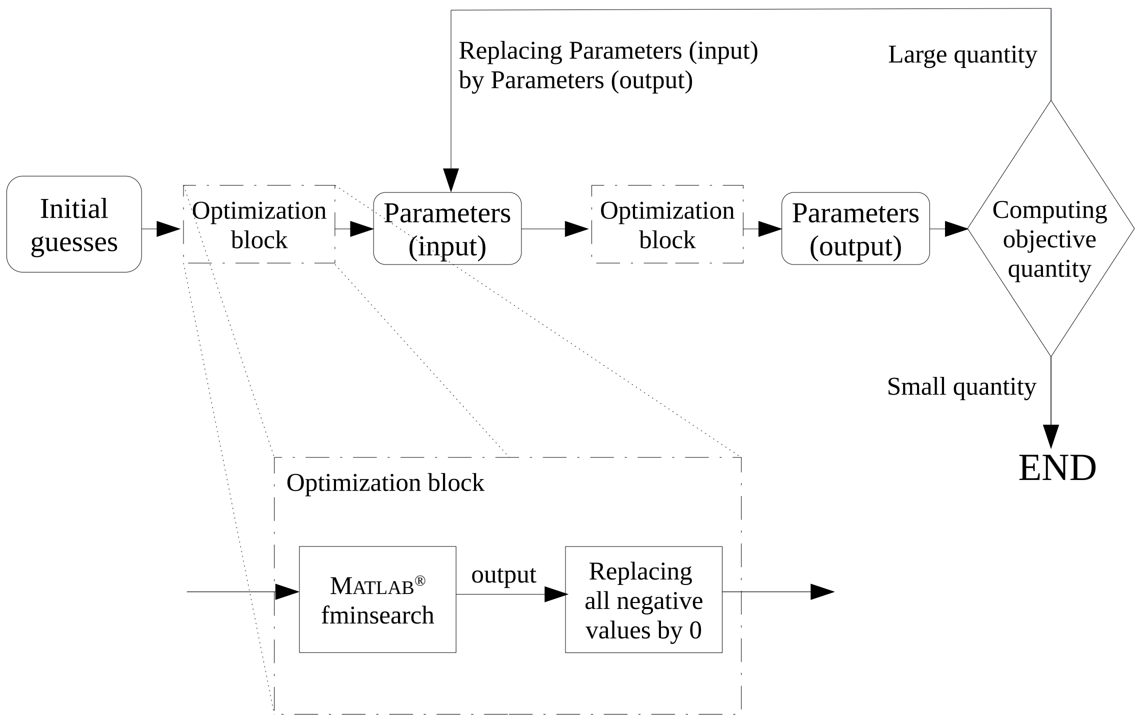

FIGURE 3.1. Flow chart of minimizing the objective quantity (3.1).

To obtain better interpolation results, it is recommended to repeat the optimization block in Figure 3.1. On the other hand, from our experience, it is better to choose the default values for `fminsearch`, while not to set larger values on `MaxFunEvals` or `MaxIter` in `optimset`.

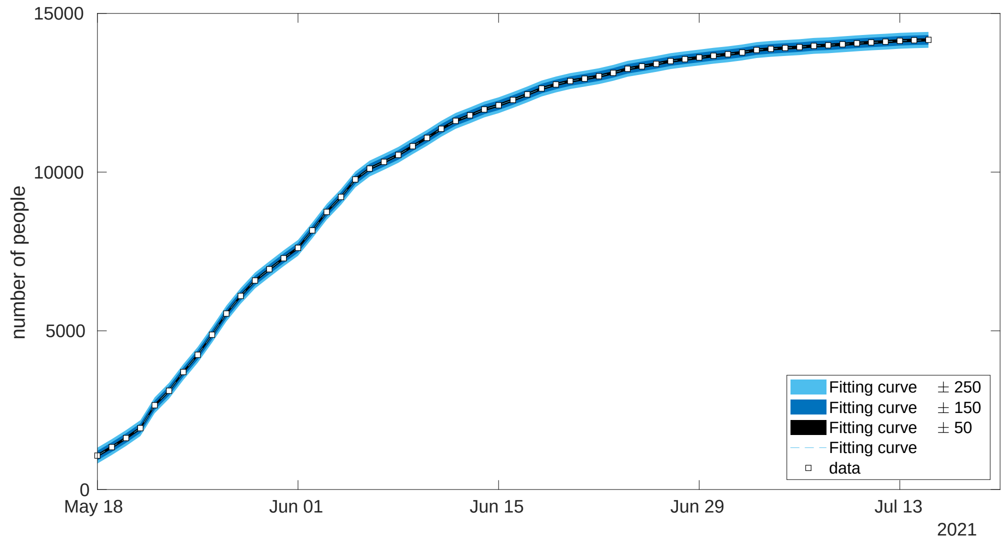

FIGURE 3.2. The fitting curve of cumulative confirmed cases based on the available data from April 22, 2021 to July 15, 2021. Here we only plot the results from May 18 to July 15 which will be used in the prediction later. We want to point out that the fitting results before May 18 is rather noisy.

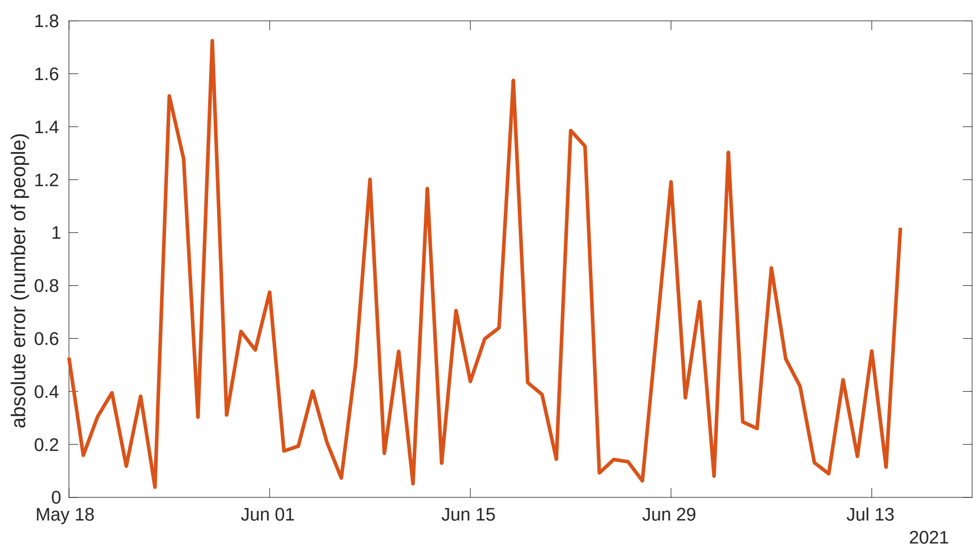

FIGURE 3.3. This figure shows the approximation error $|C_t - \hat{C}_t|$ at time t, where C_t is a minimizer of (3.1) obtained from solving the mathematical model (2.4).

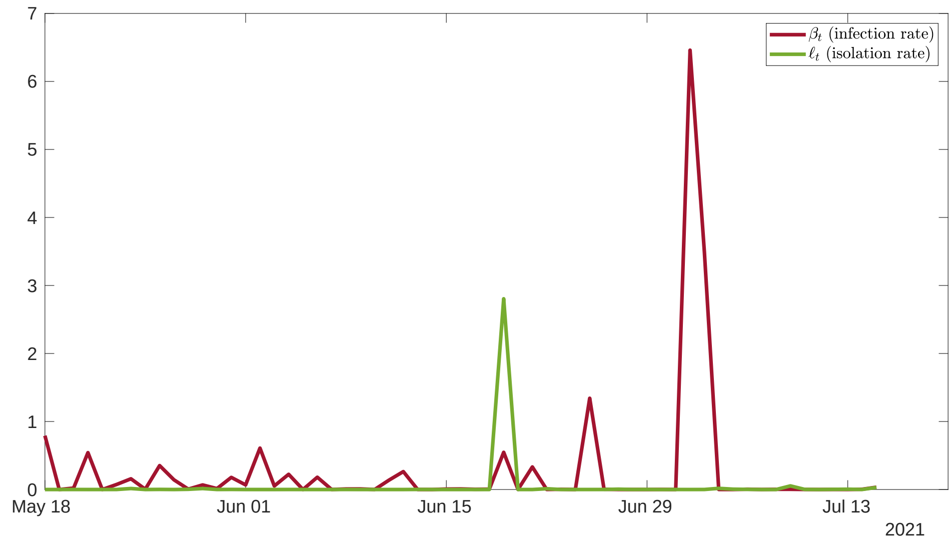

FIGURE 3.4. The interpolation results of parameters β_t and ℓ_t. The forms of the parameters are not assumed a priori. They are determined from the optimization algorithm.

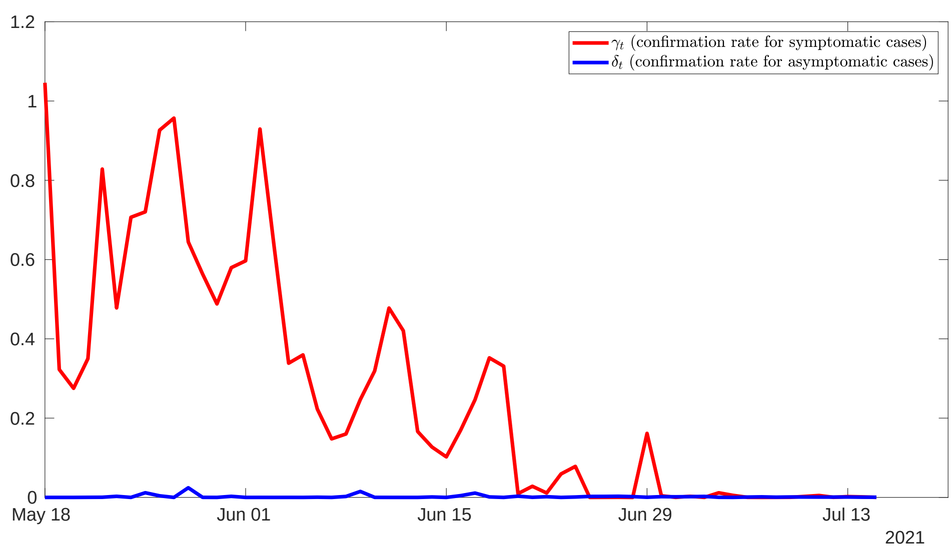

FIGURE 3.5. The interpolation results of two screen rates since, for asymptomatic cases, their Ct values (cycle threshold values) in RT-PCR tests are still in the range of being categorized as negative cases.

4. Prediction of pandemic

In this section, we will discuss the main theme of this work – forecasting of the pandemic. We want to use the data in n_{past} past days to predict the confirmed cases in the n_{future} future days. Intuitively, we assume that the following mapping is satisfied

$$\mathbb{Y}_{\text{train}} = f(\mathbb{X}_{\text{train}}), \tag{4.1}$$

where f is a nonlinear function, and

$$\mathbb{Y}_{\text{train}} = \left\{ \begin{bmatrix} C_{t+1} \\ \vdots \\ C_{t+n_{\text{future}}} \end{bmatrix} \right\},$$

$$\mathbb{X}_{\text{train}} = \left\{ \begin{bmatrix} C_t \\ \vdots \\ C_{t-n_{\text{past}}+1} \end{bmatrix}, \begin{bmatrix} \beta_t \\ \vdots \\ \beta_{t-n_{\text{past}}+1} \end{bmatrix}, \begin{bmatrix} \gamma_t \\ \vdots \\ \gamma_{t-n_{\text{past}}+1} \end{bmatrix}, \begin{bmatrix} \delta_t \\ \vdots \\ \delta_{t-n_{\text{past}}+1} \end{bmatrix}, \begin{bmatrix} \ell_t \\ \vdots \\ \ell_{t-n_{\text{past}}+1} \end{bmatrix} \right\}.$$

We usually refer the array $\mathbb{X}_{\text{train}}$ as the *input time series*, and $\mathbb{Y}_{\text{train}}$ as the *target time series*. It is impossible to determine the nonlinear function f. Therefore, in this work, we aim to approximate f using a machine-learning algorithm, called the long short-term neural network (LSTM). Here we use the TensorFlow Keras API module [AAB+15] in Python 3 language, via the Scientific Python Development Environment (SPYDER), to execute the machine learning algorithm[4]. We will sketch the idea of LSTM in next section. It is interesting to compare our model (4.1) with the model:

$$\mathbb{Y}_{\text{train}} = g(\tilde{\mathbb{X}}_{\text{train}}), \tag{4.2}$$

where g is a nonlinear function and

$$\tilde{\mathbb{X}}_{\text{train}} = \left\{ \begin{bmatrix} C_t \\ \vdots \\ C_{t-n_{\text{past}}+1} \end{bmatrix} \right\},$$

In other words, (4.2) predicts the trend of the pandemic using only the data of confirmed cases in previous n_{past} days without referring to the model (2.4). It can be seen as an unsupervised learning, while our model (4.1) is a type of supervised learning. The comparison is shown in Figure 4.1 below.

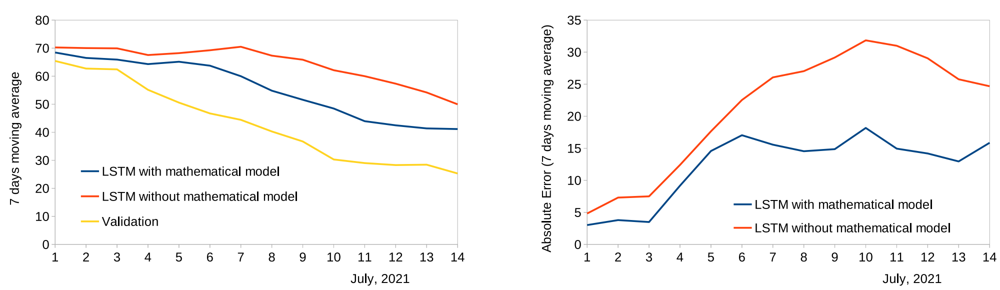

FIGURE 4.1. Comparison of models (4.1) and (4.2) using LSTM algorithm.

The result in Figure 4.1 suggests that the prediction model (4.1) works better than the model (4.2). It is noted that the LSTM algorithm is probabilistic. Any prediction curve is only one realization of the process. Therefore, we need to re-execute the program for several

[4] https://puzhaokow1993.github.io/homepage/Publications/upload/COVID19_Taiwan_Program/prediction_COVID19_export.pdf

times to find an acceptable prediction curve. From our experience of running the program, we found that model (4.1) is more stable than model (4.2).

Figure 4.1 shows the superiority of our model (4.1). Finally, we perform the prediction of the COVID-19 epidemic future trend in Taiwan from July 16 to July 27 based on our model (4.1). The prediction result is shown in Figure 4.2 and Figure 4.3. The relative error in Figure 4.3 is defined by

$$\frac{|C_t - \hat{C}_t|}{\hat{C}_t} \times 100\%,$$

where C_t and $\hat{C}_t$ represent the predicted cumulative confirmed cases (prediction) and the actual cumulative confirmed cases (validation), respectively.

From our prediction, we are quite confident of claiming that the recent outbreak of the COVID-19 pandemic in Taiwan has been brought under control and the lifting of the epidemic alert level (from Level 3 to Level 2) on July 27, 2021, is very likely. Another promising development is that people are eagerly getting vaccinated. Our lives have been disrupted recently, but we can see the light at the end of the tunnel.

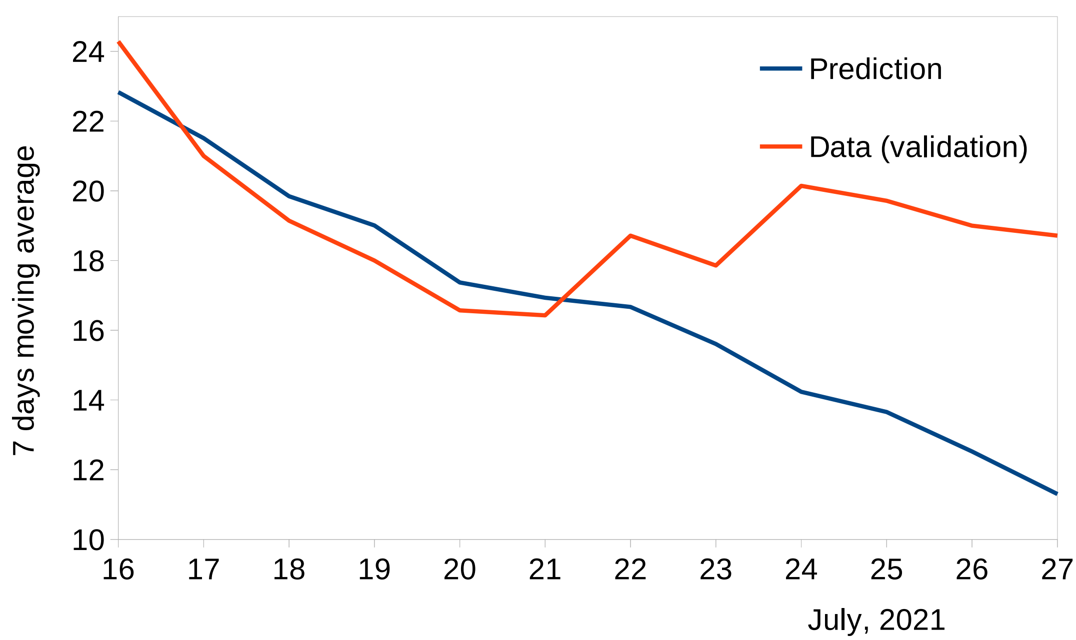

FIGURE 4.2. Prediction of the daily COVID-19 epidemic trend in Taiwan from July 16 to July 27, 2021.

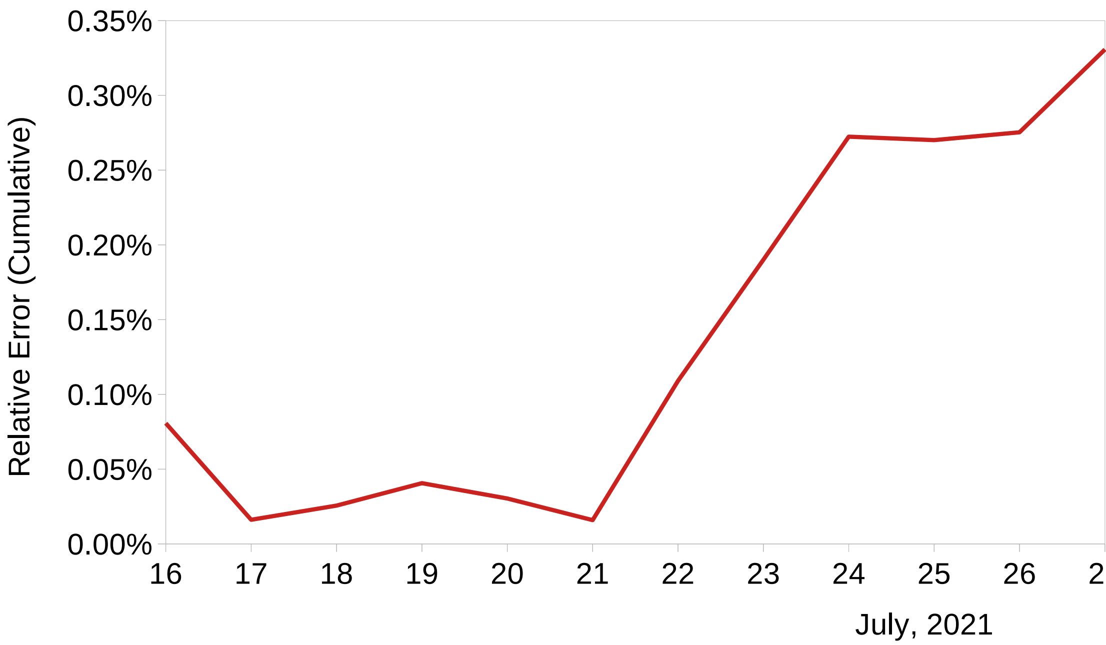

FIGURE 4.3. This figure shows the relative prediction error at time t.

5. Description of the long short-term neural network (LSTM)

In this section, we want to briefly describe the LSTM. Let $\{\boldsymbol{x}_t\}_{t\in\mathbb{N}}$, where $\boldsymbol{x}_t = (x_t^{(1)}, \cdots, x_t^{(n)}) \in \mathbb{R}^n$, be a time series. We assign the cell of state $(\boldsymbol{c}_t, \boldsymbol{h}_t)$ at each time t, where $\boldsymbol{c}_t \in \mathbb{R}^n$ represents the "memory in t-cell", and $\boldsymbol{h}_t \in \mathbb{R}^n$ represents the "hidden state of t-cell". In LSTM, t-cell learns from the following two sources of information:

- $\boldsymbol{x}_t$ (the data at time t),
- $\boldsymbol{h}_{t-1}$ (hidden state of $(t-1)$-cell),

and we denote $\tilde{\boldsymbol{c}}_t = \tilde{\boldsymbol{c}}_t(\boldsymbol{x}_t, \boldsymbol{h}_{t-1})$ the input vector. Moreover, the memory is preserved from the previous time step. Therefore, we write

$$\boldsymbol{c}_t = \boldsymbol{F}^{(t)}(\boldsymbol{c}_{t-1}) + \boldsymbol{I}^{(t)}(\tilde{\boldsymbol{c}}_t) \quad \text{for all time } t. \tag{5.1}$$

In standard LSTM, we assume that

$$\boldsymbol{F}^{(t)}(\boldsymbol{c}_{t-1}) = \boldsymbol{f}_t \circ \boldsymbol{c}_{t-1}, \quad \text{where } \boldsymbol{f}_t \in \mathbb{R}^n, \tag{5.2a}$$

$$\boldsymbol{I}^{(t)}(\tilde{\boldsymbol{c}}_t) = \boldsymbol{i}_t \circ \tilde{\boldsymbol{c}}_t, \quad \text{where } \boldsymbol{i}_t \in \mathbb{R}^n, \tag{5.2b}$$

$$\tilde{\boldsymbol{c}}_t = \sigma_c(\boldsymbol{y})|_{\boldsymbol{y}=\mathbb{W}_c\boldsymbol{x}_t+\mathbb{U}_c\boldsymbol{h}_t+\boldsymbol{b}_c}, \tag{5.2c}$$

where $\circ$ denotes the Hadamard product, that is, the elementwise product. Here, $\sigma_c : \mathbb{R} \to \mathbb{R}$ is an activation function, and usually the hyperbolic tangent function is chosen. Hence, (5.1) becomes

$$\boldsymbol{c}_t = \boldsymbol{f}_t \circ \boldsymbol{c}_{t-1} + \boldsymbol{i}_t \circ \tilde{\boldsymbol{c}}_t \quad \text{for all time } t, \tag{5.3}$$

where $\tilde{\boldsymbol{c}}_t$ is computed in (5.2c). In the machine-learning terminology, the sequence $\{\boldsymbol{f}_t\}_{t\in\mathbb{N}}$ are called the *forget gates*, and $\{\boldsymbol{i}_t\}_{t\in\mathbb{N}}$ are called the *input gates*.

Remark 5.1. If $\boldsymbol{f}_t \equiv 0$, then $\boldsymbol{c}_t$ is independent of $\boldsymbol{c}_{t-1}$, that is, the cell forgets all information at previous time. If $\boldsymbol{i}_t \equiv 0$, then $\boldsymbol{c}_t$ is independent of $\tilde{\boldsymbol{c}}_t$, namely, no input data at time t.

The hidden state $\boldsymbol{h}_t$ at time t depends only on $\boldsymbol{c}_t$. Since $\boldsymbol{h}_t$ will serve as an input in $(t+1)$-cell, hence it can be regard as the output of the t-cell. In LSTM, it is usually to define

$$\boldsymbol{h}_t = \boldsymbol{o}_t \circ \sigma_h(\boldsymbol{c}_t), \quad \text{where } \boldsymbol{o}_t \in \mathbb{R}^n, \tag{5.4}$$

where $\sigma_h : \mathbb{R} \to \mathbb{R}$ is an activation function, and normally the hyperbolic tangent function is used. The sequence $\{\boldsymbol{o}_t\}_{t\in\mathbb{N}}$ are called the *output gates*. We now provide further explanations of the gates. Assume that the vectors $\boldsymbol{f}_t, \boldsymbol{i}_t, \boldsymbol{o}_t$ all depend on $\boldsymbol{x}_t$ and $\boldsymbol{h}_{t-1}$ only. In LSTM, we usually choose

$$\begin{aligned}
\boldsymbol{f}_t &= \sigma_g(\boldsymbol{y})|_{\boldsymbol{y}=\mathbb{W}_f\boldsymbol{x}_t+\mathbb{U}_f\boldsymbol{h}_t+\boldsymbol{b}_f},\\
\boldsymbol{i}_t &= \sigma_g(\boldsymbol{y})|_{\boldsymbol{y}=\mathbb{W}_i\boldsymbol{x}_t+\mathbb{U}_i\boldsymbol{h}_t+\boldsymbol{b}_i},\\
\boldsymbol{o}_t &= \sigma_g(\boldsymbol{y})|_{\boldsymbol{y}=\mathbb{W}_o\boldsymbol{x}_t+\mathbb{U}_o\boldsymbol{h}_t+\boldsymbol{b}_o},
\end{aligned}$$

where $\sigma_g : \mathbb{R} \to \mathbb{R}$ is another activation function. In most applications, one takes σ_g a Sigmoid function. A flowchart description of LSTM is given in Figure 5.1 below.

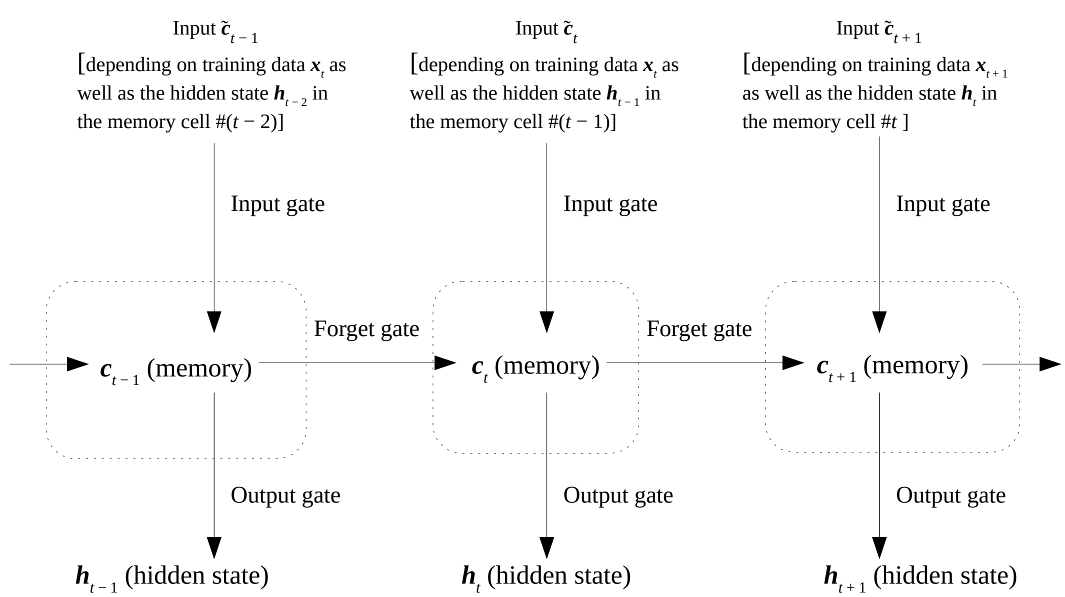

FIGURE 5.1. Long short-term memory neural network

Data availability

In this paper, we use the data of COVID-19 daily confirmed cases available from Taiwan CDC via Commonwealth Magazine.

Conflict of Interest

The authors declare that they have no conflicts of interest.

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